

HAS 222d-2009 Lecture 16

week 9

Exponential growth

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Digression on exponential growth and global population: exponential growth occurs when the growth is proportional to the population..for example, if each person has the same number of children during their life;

or, if lily pads grow on the surface of a pond by reproducing at a constant rate...the effect is very slow growth, almost invisible, followed by explosive growth until the pond is covered. What sort of physical processes lead to exponential growth? The key is that the slope of the curve e^{At} is Ae^{At} ... slope is proportional to value. If t is time, this says that the rate of change of the curve is proportional to its value. If each human has 1 child, and life span is unchanging in time, then over time population will follow exponential growth, with a *growth rate* a : the population increases by a factor e in time $1/A$. Any process like lily pads that, for some time, increase in proportion to their numbers, will be exponential.

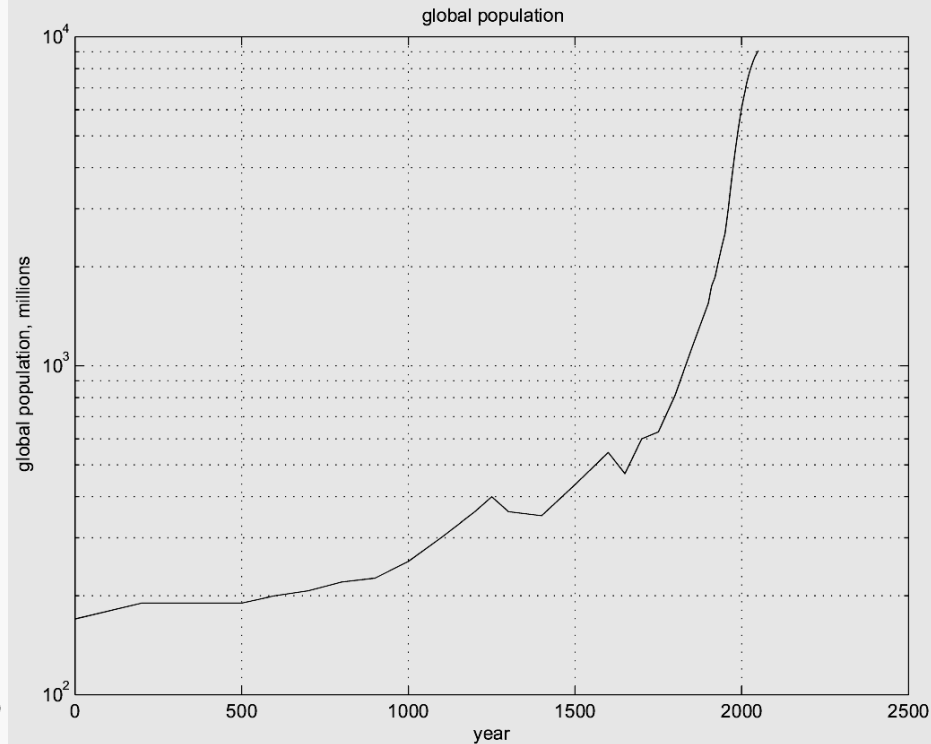
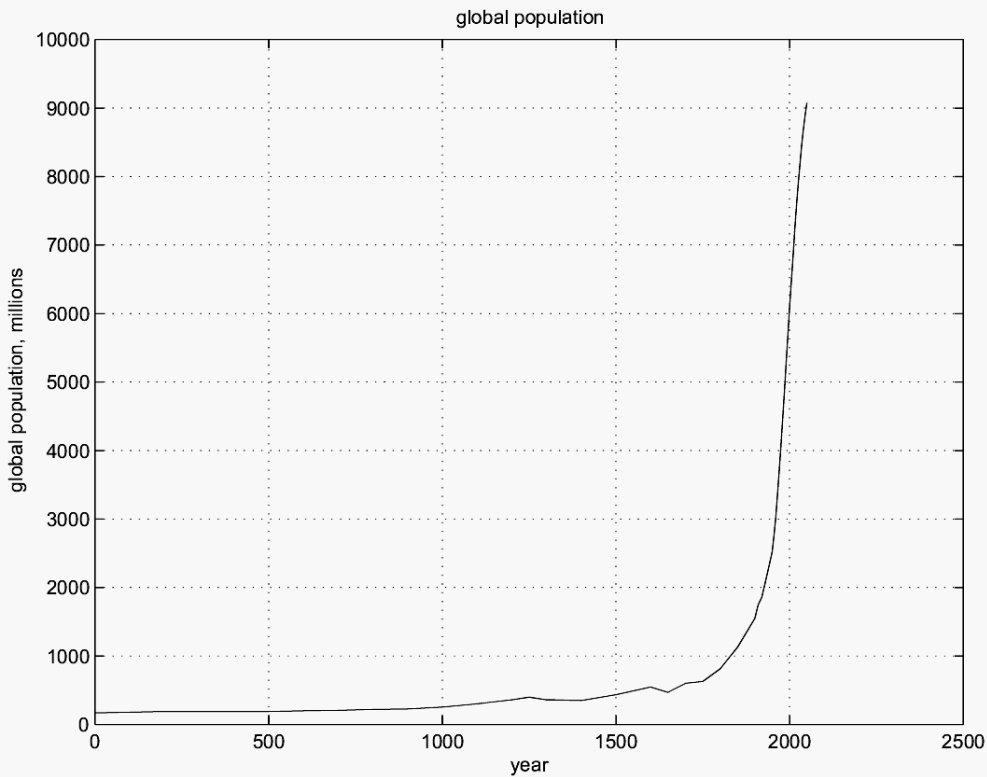
Is global human population growing exponentially?

exponential growth?

linear and semi-log plots of global population against year

An exponential curve appears as a straight line on a semi-log plot
[$\log(\text{population})$ plotted against time, where the vertical axis has equal intervals for each power of ten (or power of e)]. Apparently global population is 'hyper-exponential'.

- See the separate notes posted on the class website too.



- notice that $y = d e^{at}$ and $t = (1/a) \log(y/d)$ are complementary relations (the same curve turned round): the exponential curve rises faster than any power of t , and the log curve rises slower than any power of y . (log here is log to the base e , $e = 2.718\dots$).

If you plot an exponential curve on a distorted plot, with axes $\log(y)$ and t , it is a straight line with slope a and intercept $\log(d)$ (the value at $t=0$). This is very useful as a way to look at data that ranges widely, from very big to very small.

Exponential growth is more extreme than 'algebraic' growth. If you have a power law, it grows more slowly than exponential:

$$y = g t^n$$

can also be plotted with distorted axes, $\log(y)$ and $\log(t)$. Here, the power law will appear as a straight line with slope n and intercept $\log(g)$, because

$$\log(y) = \log(g) + n \log(t)$$

These are good empirical ways to assess growth rates and distinguish exponential behavior and algebraic behavior.

Doubling time: 'the rule of 70'

- $P = P_0 \exp(\alpha t)$

we speak of the 'e-folding time', the time for P to increase by a factor e ($=2.718$). More natural is to consider the doubling time: P goes from

	P_0
to	$2P_0$
when	$\exp(\alpha t) = 2$
or at	$\alpha t = \log_e(2) = .6931,$
that is at time	$t = .6931/\alpha$

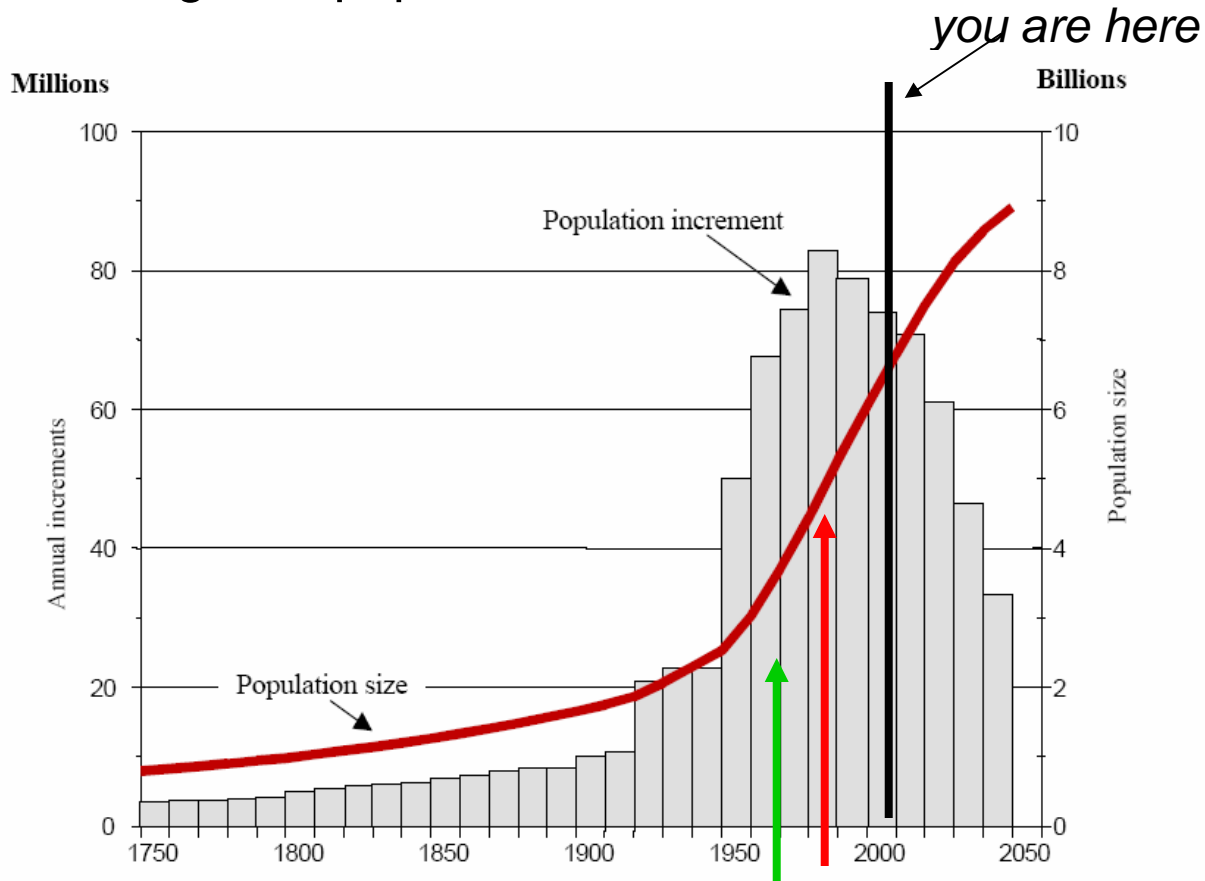
- α is the 'growth rate' with units time^{-1} . If α is expressed in percent rather than as a decimal value, then $69.31/\alpha$ is the doubling time. A good approximation is:

doubling time in years = 70 divided by the growth rate in %

- With a credit card loan at an 18% interest rate you will pay a bit more than double the purchase price if the loan is for 4 years, paid off all at the end.

Global population and its rate of change, per year

US population: 298.7 million as of May 2007
global population: 6.7 billion “ “ “

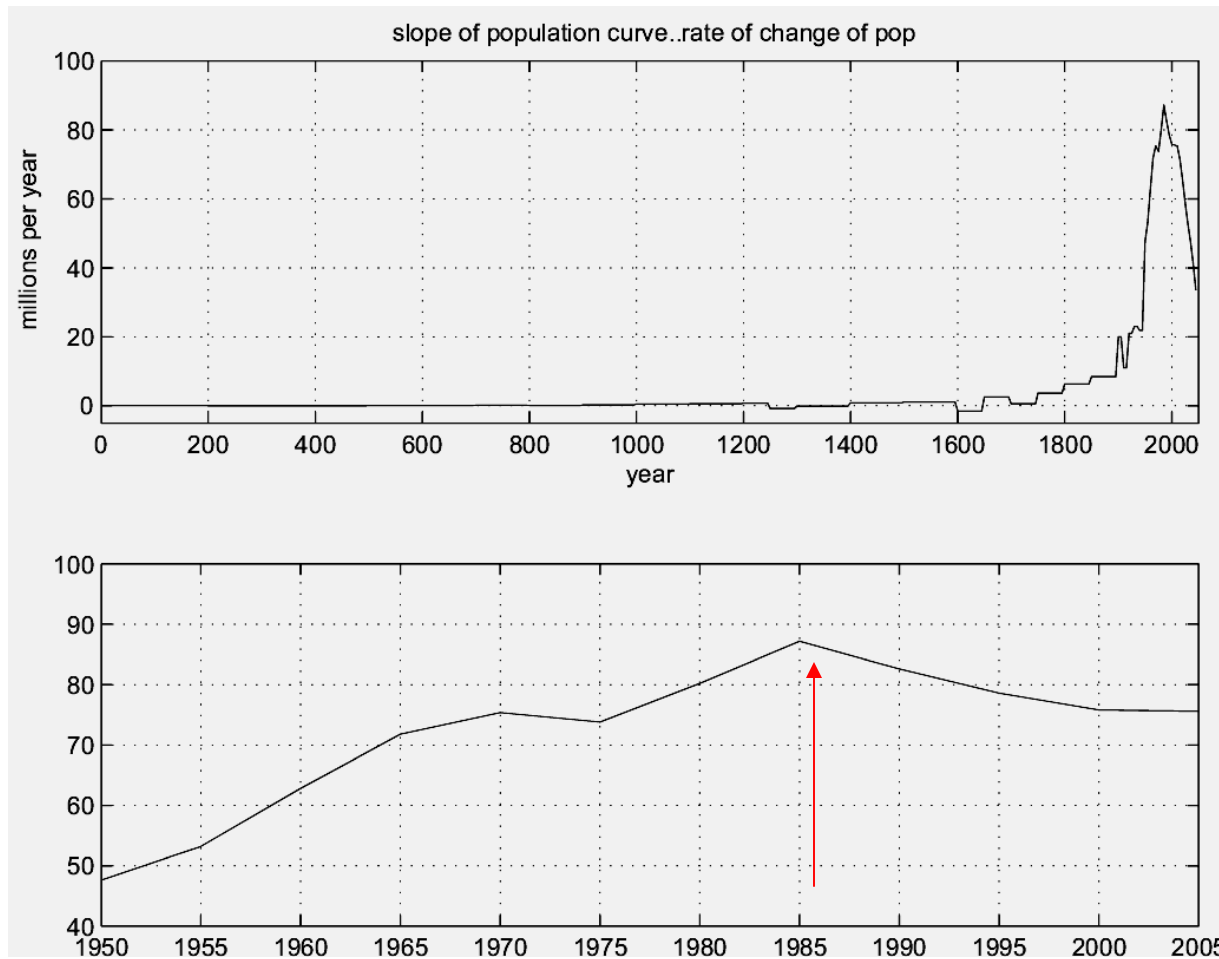


Source: United Nations Population Division.

largest %
growth
rate

largest
absolute
growth
rate

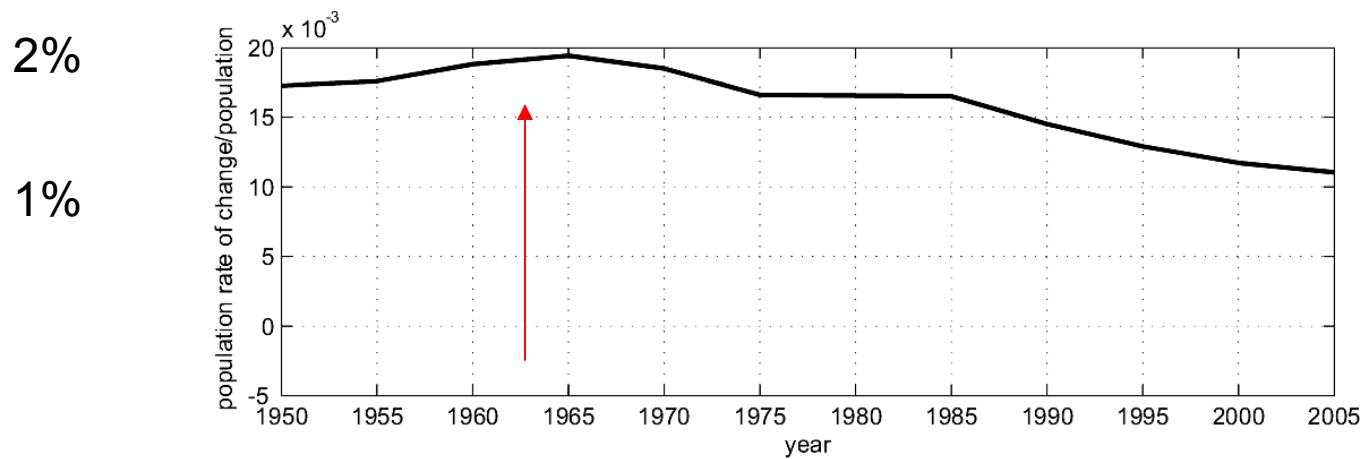
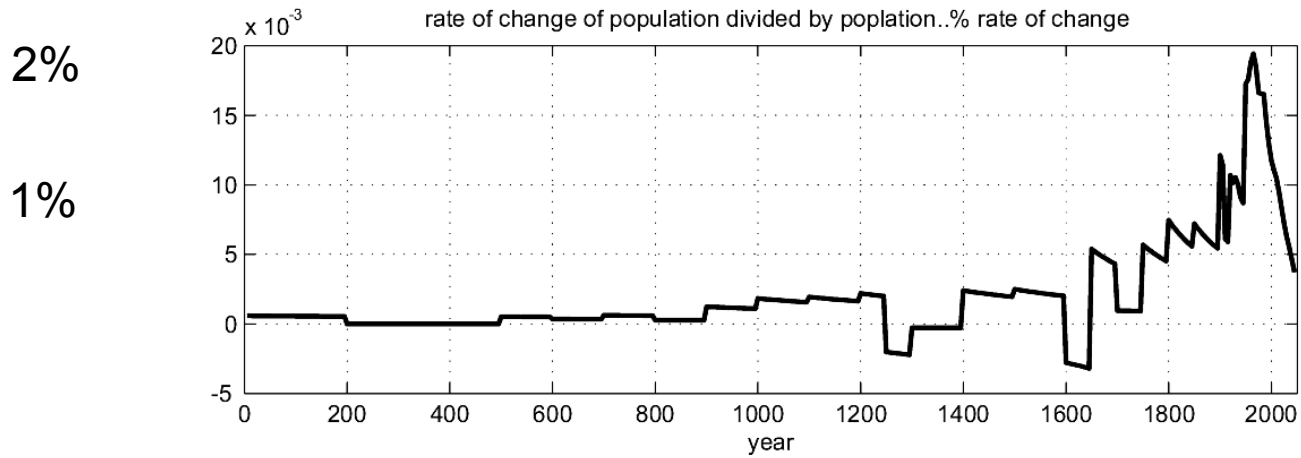
slope of population curve...rate of growth of population (millions per yr)



zoomed

peak in 1985

rate of growth of population (% per year)



peak in 1963-65

doubling time of population = $69/\text{growth rate}$
(growth rate in % per year)

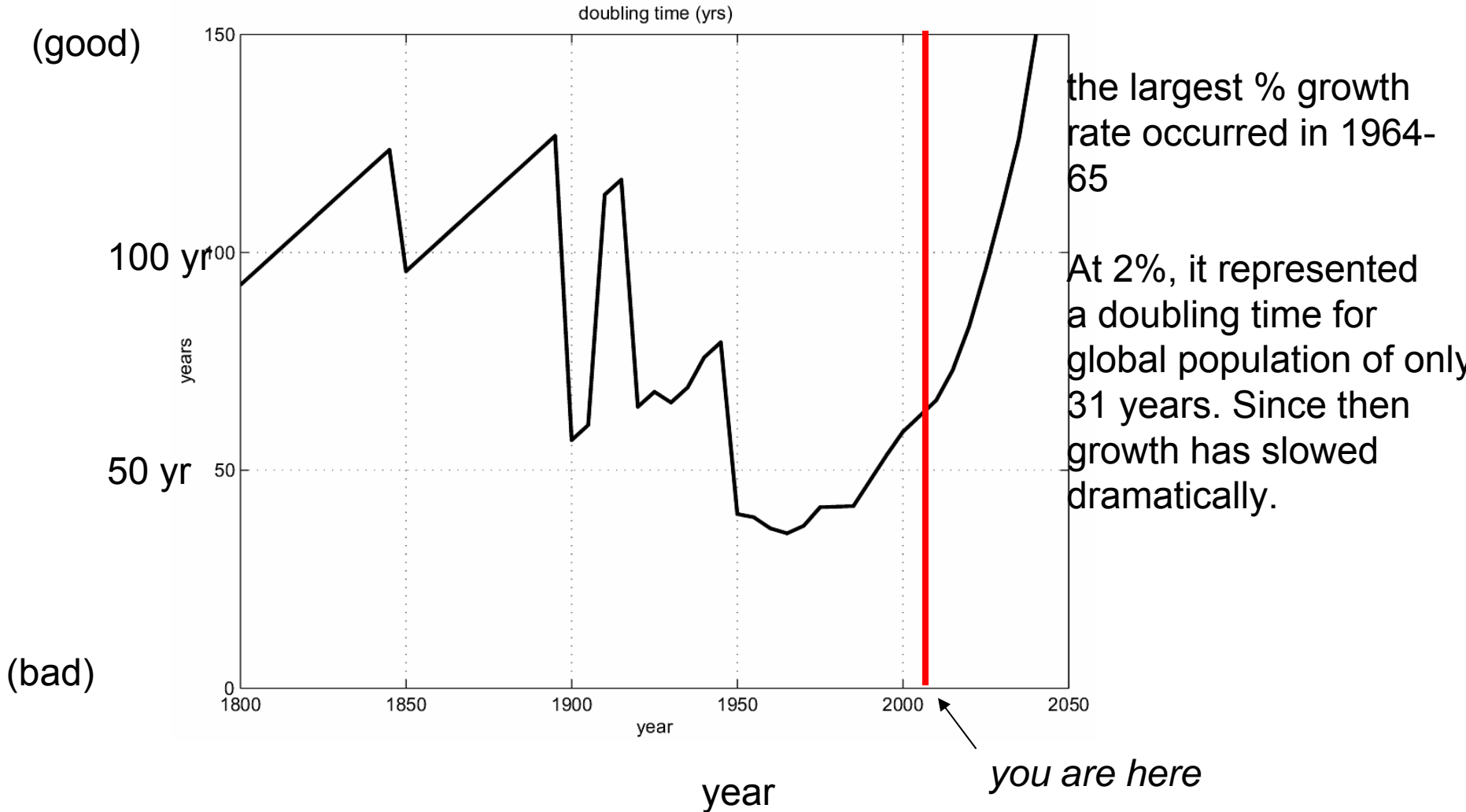
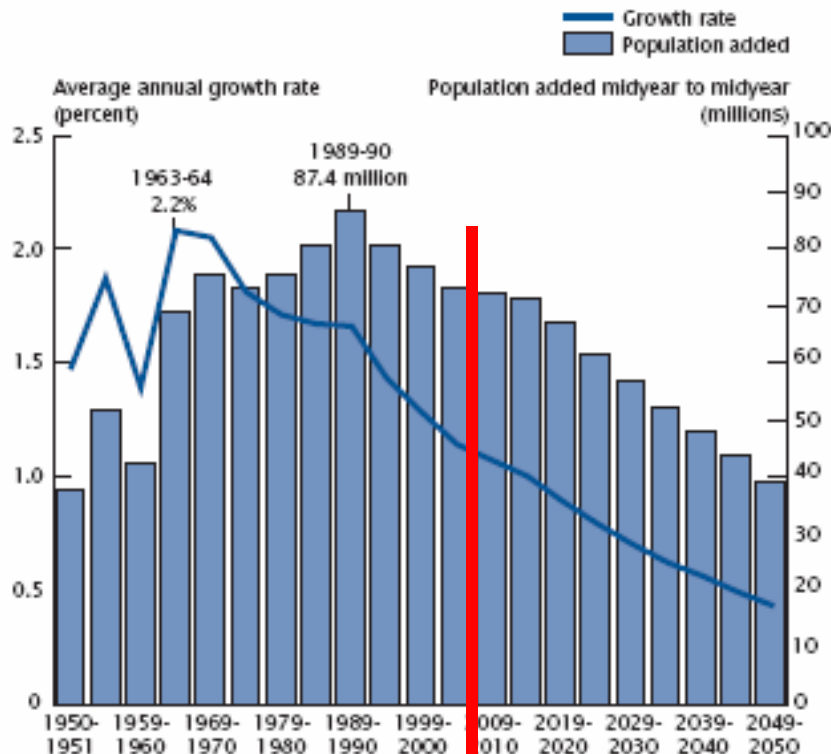


Figure 4.
**Annual Additions and the Annual Growth Rate of
 Global Population: 1950-2050**
**By 2002, the pace of global population growth
 was on the decline.**



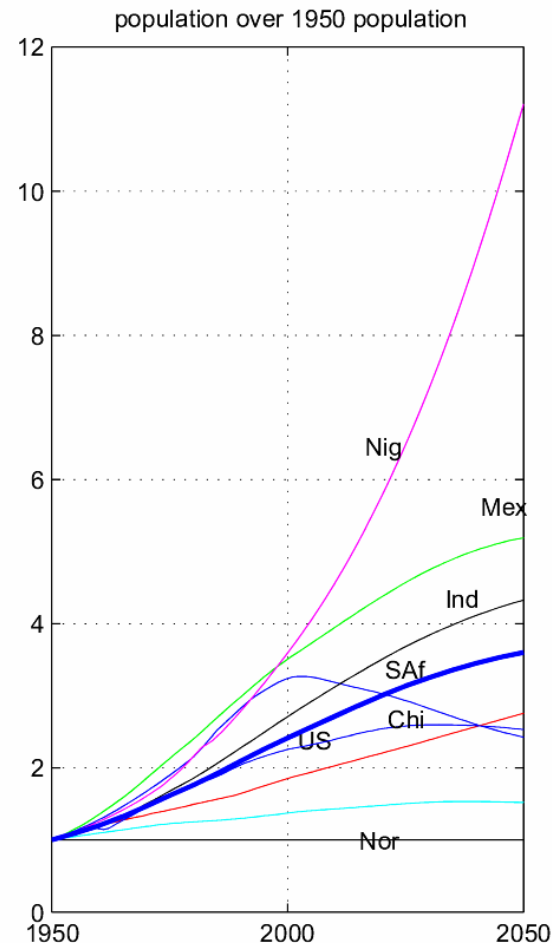
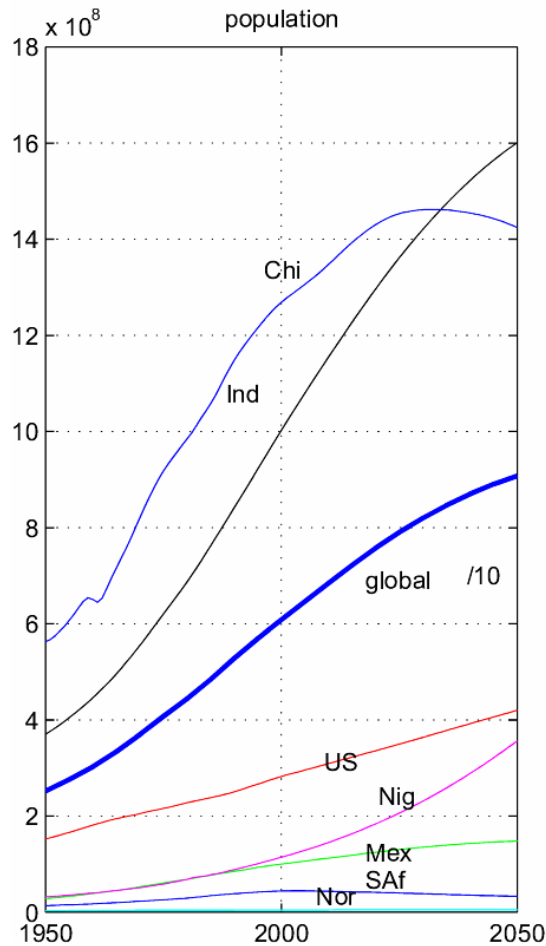
Source: U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.

The Single Largest Annual Increase in World Population Is Estimated to Have Occurred in 1989-90

As Figure 4 shows, however, the average annual rate of growth for the globe's population peaked in 1963-64.¹ Then, in 1989-90, the absolute annual increase to global population reached its apex and began to decline. According to Census Bureau projections, the pace of global population growth will continue to decline into the foreseeable future.

¹ In Figure 4, the dip in population growth during 1959-60 was due to a severe famine in China.

Population (left) and population/1950 population) for China, India, US, Nigeria, Mexico, South Africa, Norway and global. Projection to 2050.





Ladybugs and aphids

As long ago as 1789, Thomas Malthus studied the nature of population growth in Europe. He claimed that population was increasing faster than food production, and he feared eventual global starvation. Of course he could not foresee how modern technology would expand food production, but his observations about how populations increase were important. Population grows geometrically (1, 2, 4, 8 ...), rather than arithmetically (1, 2, 3, 4 ...), which is why the numbers can increase so quickly. Look to Nature: are we that different from other animals and insects? In Oregon a few years ago there was a huge increase in the ladybug population, spurred by an unusual supply of their favorite food, aphids. But, just as rapidly, the ladybug population soon crashed when they overran the food supply.

Notice how the global population hit a very high growth rate of 2% in 1965, then declined.

Note that exponential growth, $y = d e^{at}$ which follows this pattern of doubling every $69/a$ time units, is more extreme than “algebraic growth”, compare with

growth function	$y = bt^2$	or $y = ct^3$	rate of change of y	rate of change of y divided by y
$y = de^{at}$			ay (exponential growth)	a
$y = bt^2$			$y^{1/2}$ (t^2 growth)	$2(b/y)^{1/2}$
$y = ct^3$			$y^{2/3}$ (t^3 growth)	$(3c/y)^{1/3}$

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Now, if the global population hit a very low growth rate of 2% in 1965, the population would double every 35 years.

Note that exponential growth, $y = ce^{at}$, which follows this pattern of doubling every 69/a years, is much more than "algebraic growth", compare with $y = bt^2$ or $y = ct^3$.

$y = de^{at}$

$y = bt^2$

$y = ct^3$

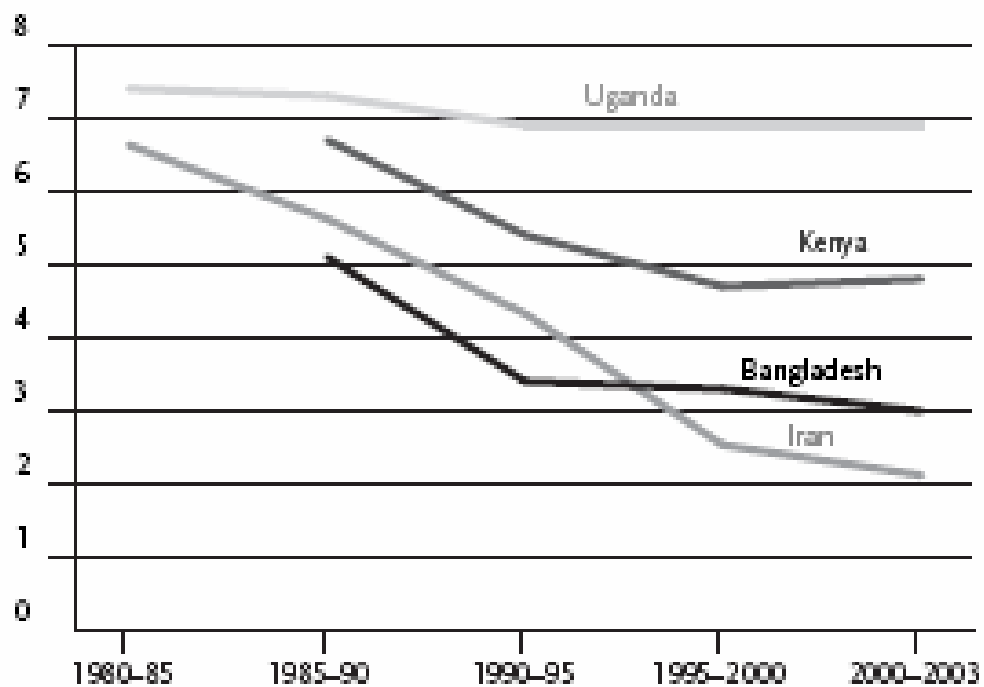
rate of change of y
ay (exponential growth) a

$y^{1/2}$ (t^2 growth) $2(b/y)^{1/2}$

$y^{2/3}$ (t^3 growth) $(3c/y)^{1/3}$

- 1. World population crossed the six billion threshold in 1999.
- 2. World population is projected to cross the 7 billion mark in 2013; the 8 billion mark in 2028; the 9 billion mark in 2054. World population nearly stabilizes at just above 10 billion after 2200.
- 3. It has taken just 12 years for the world to add this most recent billion people. This is the shortest period of time in world history for a billion people to be added.
- 4. World population did not reach one billion until 1804. It took 123 years to reach 2 billion in 1927, 33 years to reach 3 billion in 1960, 14 years to reach 4 billion in 1974 and 13 years to reach 5 billion in 1987.
- 5. **The highest rate of world population growth (2.04 per cent, 34 year doubling time) occurred in the late 1960s. More recently the rate (1995-2000) was 1.31 per cent (53 year doubling time), and in 2005 was close to 1.0%. (69 year doubling time)**
- 6. **The largest annual increase to world population (86 million) took place in the late 1980s; the current annual increase is 78 million. Note the difference**
- 7. Of the 78 million people currently added to the world each year, 95 per cent live in the less developed regions.

Lifetime births per woman

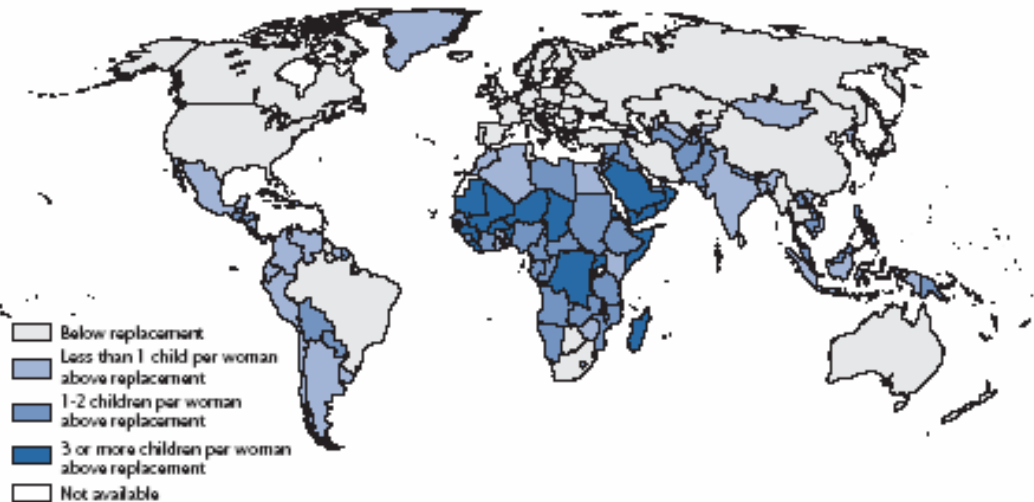


Source: Demographic and Health Surveys and United Nations Population Division.

http://www.prb.org/pdf05/05WorldDataSheet_Eng.pdf

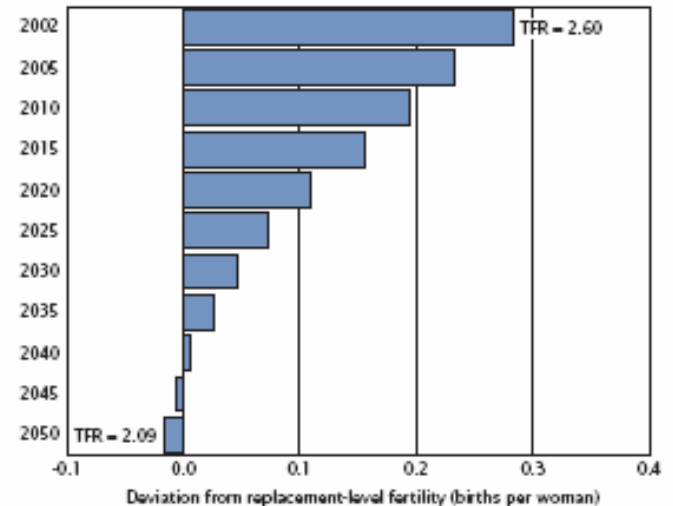
surprisingly, many countries have birth rates below the steady-state replacement level (about 2.4 children per couple)

Figure 9.
Total Fertility Rate Relative to the Replacement Level for Each Country Across the Globe: 2002
In 2002, fertility levels were highest in the countries of Sub-Saharan Africa and the Near East, and lowest in the more developed countries.



Source: U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.

Figure 10.
Global Fertility Levels Relative to Replacement Level: 2002-2050
The level of global fertility is projected to drop below replacement by mid-century.



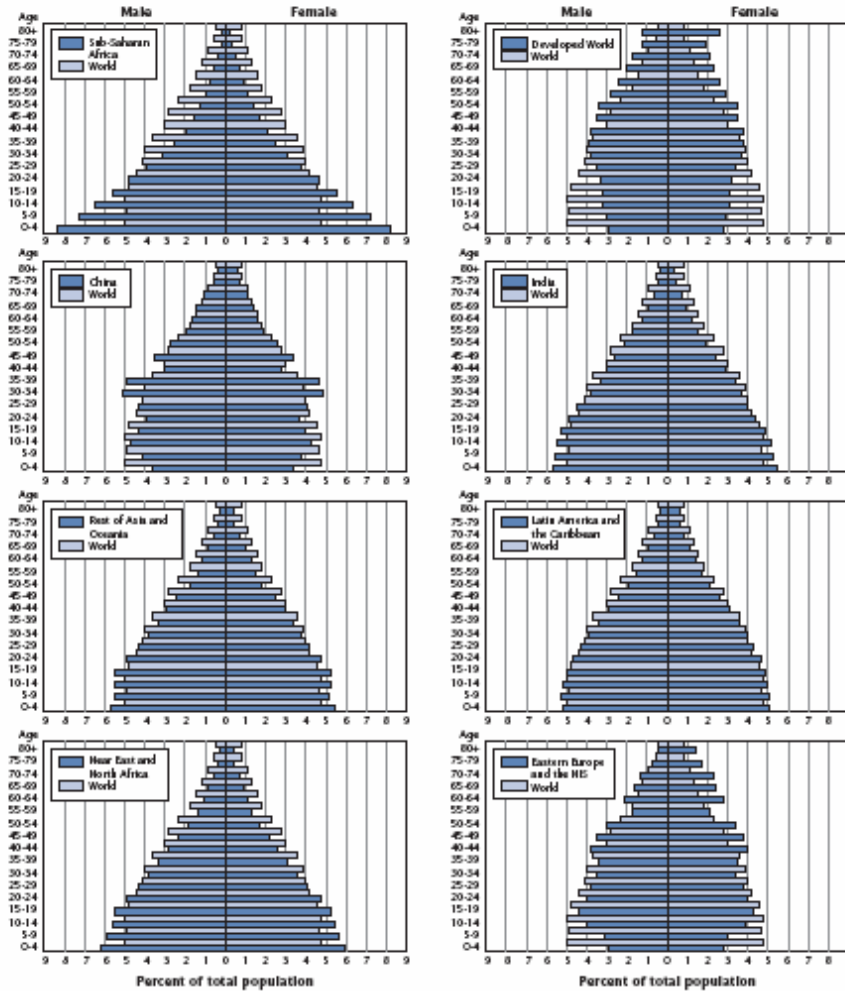
Note: Global total fertility rates underlying figure 10 were derived by calculating weighted age-specific fertility rates from country-and-age-specific births and numbers of women.
 Source: U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.

Figure 16.
Net Migration Rates Around the Globe: 2002
The effect of net migration on population size in 2002 was small to moderate for most countries.



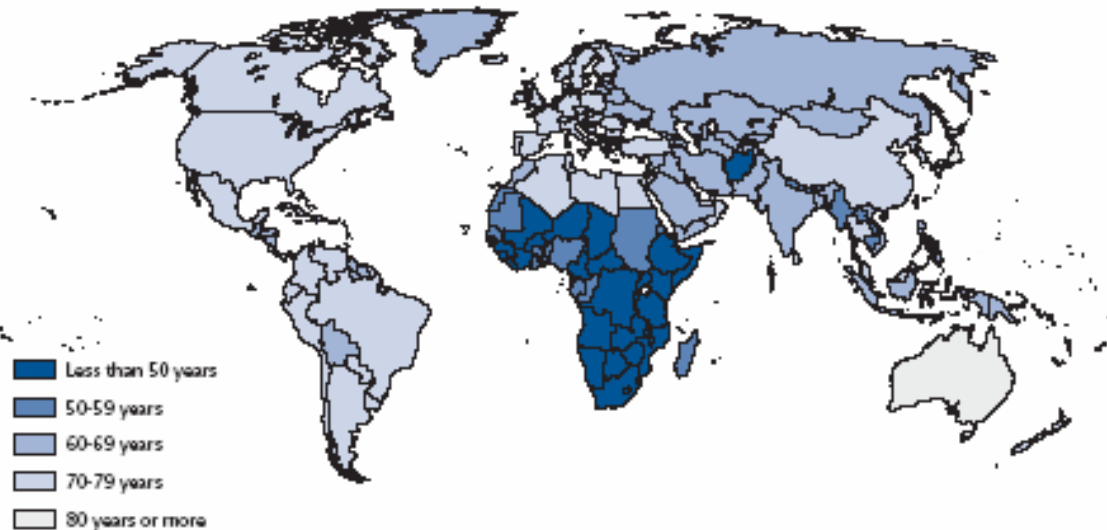
Source: U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.

Figure 19.
Population Pyramids by Region and Selected Countries: 2002
The age-sex compositions of Sub-Saharan Africa and the developed world
reflected the remarkable variation across the globe.



Source: U.S. Census Bureau, International Programs Center, International Data Base.

Figure 12.
Life Expectancy at Birth Across the Globe: 2002
Sub-Saharan African countries had by far the lowest life expectancy at birth of any region on the globe in 2002.



Source: U.S. Census Bureau, International Programs Center, International Data Base and unpublished tables.