GFD-1 Winter 2004 Final Exam **[total score: 10 points]**
Monday 15 March 10.30-12.20 ATG 610
Books and notes may be used

1. (Short answers, say a short paragraph each) [1 point each]

a. •Describe how the warm ocean heats the atmosphere: what kinds of heat transport are involved?

b. Suppose that a broad ocean current in a single-layer, constant-density rotating ocean flows eastward with constant velocity (U_0, 0). It flows from deep water into shallower water (the bottom slopes gradually up to the east). Assuming the zonal velocity u is much greater than the meridional velocity v, The fluid is confined in a channel with walls parallel to the x-axis

•describe the change in velocity profile u(y) that we expect to occur as it encounters the slope.

c. •Does the compressibility of a fluid raise or lower its buoyancy frequency? Explain why.

d. In Ekman boundary layer theory we derived the expression

$$\vec{\tau} = -\rho \vec{M} \times \vec{f}$$

[$\vec{\tau}$ is the horizontal stress exerted by the top or bottom boundary on the fluid, $\vec{M}$ is the vertical integral of the horizontal velocity (the volume transport in the Ekman layer, minus that due to the basic interior velocity), and $\vec{f}$ is twice the Earth's rotation rate.

• Recall the wind-stress driven channel flow worked out in lectures. Describe qualitatively (using a sketch), the movement of fluid in the y-z plane when this single layer of fluid is forced by a surface 'wind' stress in the x-direction and by an Ekman layer at the bottom (where u=0, v=0 at the bottom).

[This 'viscous overturning circulation' gives the same result for the velocity (u,v) of the interior flow as the linear bottom-drag and body force did, in the lectures]

e. Consider an ideal gas confined in a balloon, which changes its volume. In the 1^{st} law of thermodynamics there is a balance between change in internal energy, heat input and work done by pressure forces.

• Describe from a *molecular* standpoint what these three terms represent. This is an ideal gas in which the energy of the molecules is entirely kinetic, and they experience elastic collisions.

3. [3 points] Thermal wind

Suppose the pressure field in the atmosphere or ocean is given at two different heights,

$$p(z = z_1) = Ax$$

$$p(z = z_2) = Bx + Cy$$

where A,B, C are constants. Assuming hydrostatic, geostrophic dynamics,

•calculate the density field, (the average of $\rho'(x,y)$ between levels z_1 and z_2). The density is $\rho = \rho_0(z) + \rho'(x,y)$. There is a uniform background stratification, $N^2 = (-g/\rho_0)\, d\rho_0/dz$.

•Calculate the geostrophic velocity field.

•Make a sketch showing where the fluid is warm and where it is cold (if we take the density to a function of temperature only).

•On this sketch show the relation between the velocity vector at the two levels, and the horizontal density gradient.

• For the more complex pressure field,

$$p(z = z_1) = P \cos k(x - a)\ \cos ly$$

$$p(z = z_2) = P\ \cos k(x + a)\ \cos ly$$

where P = constant, sketch (and calculate if you have time) the geostrophic velocity, the relation between the vector velocity and the horizontal density gradient, and the regions of warm and cold fluid.

- If you have time, relate this pattern to the pattern of the Icelandic Low, its variation with height, and the presence of cold and warm air.

4. Coriolis [2 points]

If we have motion varying like exp(-iσt) in time, the linearized horizontal momentum equations for a single layer of fluid of constant depth (no free surface) become

$$-i\sigma u \; - \; fv = -\frac{1}{\rho}\, p_x$$

$$-i\sigma v \; + \; fu = -\frac{1}{\rho}\, p_y$$

This neglects the advective acceleration terms. The physical variables are the real parts of u exp(-iσt), etc.

- Solve for u in terms of p_x and p_y (eliminating v). Give a physical interpretation of this expression for cases $\sigma \gg f$ and $\sigma \ll f$. Note this is not a differential equation problem, just an algebraic manipulation.
- What does this result tell you about the velocity field in a long hydrostatic gravity wave with rotation, where the free surface elevation is η = Real A exp(ikx-iσt)?

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formula: cos(A+B) = cos A cos B – sin A sin B