

From rhines@ocean.washington.edu Wed Jan 28 13:55:01 2004
 Date: Wed, 28 Jan 2004 13:50:13 -0800 (PST)
 From: Peter Rhines <rhines@ocean.washington.edu>
 To: carson2@u.washington.edu, beth4cu@u.washington.edu,
 kmartini@u.washington.edu, zarnet@u.washington.edu,
 jadam@u.washington.edu, bbale@amath.washington.edu,
 semaj@u.washington.edu, scavallo@u.washington.edu,
 starlush@u.washington.edu, brianal@u.washington.edu,
 lharris4@u.washington.edu, michaelh@u.washington.edu,
 yij@u.washington.edu, tkubar@atmos.washington.edu,
 nlindber@u.washington.edu, nelslindberg@hotmail.com,
 rnich@u.washington.edu, bpontare@u.washington.edu,
 steine@u.washington.edu, park@hydro.washington.edu
 Cc: eleanor@ocean.washington.edu, dank@atmos.washington.edu,
 Peter Rhines <rhines@ocean.washington.edu>
 Subject: FAQ's on PS-2.4

[This will also be on the GFD1 website.]

Problem 4 on Problem Set 2 is meant to show exactly how the pressure field looks for an interesting flow, with and without rotation. In the world of GFD we so often make the geostrophic approximation, yet the small error (the difference between the true pressure and geostrophic pressure) is what makes the fluid accelerate and decelerate in this flow: it's important, and Bernoulli tells us just what it is.

The first thing to do is calculate the pressure $PNR(r,\theta)$ or $PNR(x,y)$ using Bernoulli for the case $f=0$ (PNR means non-rotating pressure, P). This is

$$PNR + \frac{1}{2}\rho(u^2 + v^2) = [PNR + \frac{1}{2}\rho U^2] \Big|_{x \rightarrow -\infty}$$

where the righthand side is evaluated far upstream, and the lefthand side is evaluated anywhere. $(U,0)$ is the uniform eastward flow far upstream.

Make a good plot and note the maximum and minimum values of p and where they are. Let us call this pressure PNR for non-rotating. Understand the need for the highs and lows, owing to the acceleration of fluid particles.

Next consider the effect of rotation. If the velocity field is the same (which it is in this idealized 2 dimensional flow), then we know that the new term, the Coriolis force, $2\vec{\Omega} \times \vec{u}$, is perpendicular to the

velocity vector, \vec{u} and hence it can be balanced by a extra pressure field whose gradient is perpendicular to \vec{u} , say PR for rotating contribution to pressure.

Thus $P = PNR + PR$. We know the isobars of PR are parallel to streamlines, whereas the isobars of PNR are not. Thus PR is some function of ψ , $PR = g(\psi)$ say. Now we go far upstream to find out what that function is.

Bernoulli's equation is unchanged by rotation (because it comes from

integrating the momentum equation along streamlines, in the direction of zero Coriolis force). So there is no new information there. We already know that P_R is constant along streamlines, so it adds to both sides of Bernoulli's equation.

Far upstream, $x \rightarrow -\infty$, the y -momentum equation tells us that

$$P = P_{NR} + P_R = P_{NR} + \text{some function of } y. \text{ (all evaluated at } x \rightarrow -\infty)$$

What is that far-upstream dependence of P_R on y (from geostrophic balance)? The result is that the new pressure $P_R + P_{NR}$ is high on the south side and low on the north side of the flow. At this point I would suggest first sketching the effect of the new pressure term, how the highs and lows are shifted. P_R is constant along each streamline but varies across streamlines.

Finally writing down the expression for P_R can be done by thinking about this upstream boundary condition. Establish the relation between P_R and ψ far upstream, and it will be the same relation everywhere in the flow: this establishes the function $g(\psi)$ which we did not know before using the boundary condition. Since we know ψ as a function of r, θ or x, y we then know the total pressure P everywhere. A Matlab contour plot of this would be nice.

-Peter R.

Peter B. Rhines
Prof. of Oceanography and
Atmospheric Sciences
Box 357940
University of Washington
Seattle, Washington, 98195
206-543-0593 (319 OSB..office)
206-685-3548 (107 OSB..GFD lab)
Web sites:
www.ocean.washington.edu/research/gfd/gfd.html
and
www.ocean.washington.edu
(=> Faculty => Rhines)
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