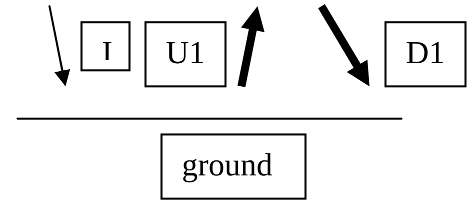
GFD-1 Problem Set 1 OC-512/AS-509

out: 12 Jan 2001
back: 21 Jan 2001 (Wednesday)

1. The *greenhouse 'blanket'.* Carry out the greenhouse calculation in Gill, except use two plates of glass (thick lines on the diagram below), each of which absorbs (and re-radiates) a fraction γ of the long-wave infrared radiation reaching it, yet passes short-wave visible light with no reflection or absorption. Show that the increase in surface temperature at the ground can be greater than for a single glass plate absorbing a fraction γ of the incident radiation, even if $\gamma=1$ in both cases. Recall that the outgoing longwave radiation on one side of a pane of glass is equal to the partially transmitted longwave plus ½ the absorbed longwave radiation (½ because the absorbed radiation is re-radiated equally above and below). Set up the equations as in Gill p9, and find the upward longwave radition in the lowest layer in terms of I, the incident short-wave solar radiation; hence if each pane of glass is a perfect absorber of IR ($\gamma=1$), compare the temperature at the ground with the temperature with 1 pane of glass and with none.

(Hint, it seems easier to do this problem as sketched below, with just a single upward (U) and a single downward (D) longwave component in each layer (rather than defining extra components like B in Gill's fig. 1.3. Then for example the equation for the upward longwave radiation in the middle layer is $U_2=U_1(1-\gamma) + \frac{1}{2}\gamma U_1 + \frac{1}{2}\gamma D_2 =$ partially transmitted upward flux + re-radiated upward flux + re-radiated downward flux. We can see that the origin of the extra trapping by the 2d pane of glass is the downward component D_2 which is absent with a single pane of glass).



2. *Hydrostatic pressure.* In a fluid at rest, the vertical momentum equation in Cartesian coordinates is $\partial p/\partial z = -\rho g$. In words, the pressure at a point is equal to the weight of the fluid overhead (per unit area), plus whatever the pressure is at the top of the fluid.

Calculate the total force (pressure x area) on the bottom of a Coke bottle and show that is *not* equal to the total weight of the fluid in the bottle. Explain.

3. *What is pressure?* A point mass with velocity +U in the x-direction and mass m hits a rigid wall and rebounds with velocity -U. Show that the time integral of the force exerted by the point mass on the block is 2mU (this time-integrated force is known as the 'impulse'). If instead of rebounding elastically, the particle sticks to the wall, what is the time-integral of the force it exerts on the wall? In the first example the kinetic energy of the particle stays the same; what happens to the kinetic energy in the second example?

If many such particles hit a wall, say their density in space is n particles per cubic meter, so that they continually arrive and bounce off the wall, what is the time-averaged force they exert on the wall? [assume for simplicity they are equally distributed in space and hit the wall at equal time intervals].

This is a model of the pressure force exerted on a wall by 'point-mass' molecules of a dilute gas. The model shows that the pressure force is proportional to the mass density of the particles which we call density, and a quantity proportional to the kinetic energy (KE) of the molecules, which in turn we call 'temperature'. Pressure is also an expression of the *momentum flux* of the particles which is proportional to u^2.

{*Further discussion*: after the model is generalized to 3 space dimensions we find (Batchelor, Intro. to Fluid Dynamics, 1.7)

$$\text{force/area} = p = 1/3\ \rho <u^2>$$

where the brackets < > indicate the average value and u is the speed (=magnitude of the velocity). This becomes $p = nkT = (k/m)\rho T = \rho RT$ where: $\rho = nm$ is the average density in kg m^{-3}, $k = 1.381 \times 10^{-23}$ Newton $m/^0C$ is Boltzmann's constant and R is the gas constant defined as k/m . Generally the temperature of a gas is proportional to the translational kinetic energy of molecules plus rotation and vibration energies of the molecules. But, in the simplest case of point masses with no rotation or vibration, the average KE of a molecule is just $½\ mu^2 = 3/2\ kT \equiv 3/2\ mRT$. Thermodynamics texts often avoiding telling us just what temperature is!}

4. Bobbing buoys. A spar buoy is a thin cylinder (like a log) which is floating vertically. If its height is H, cross-sectional area is A and mass density is ρ_B,

- calculate the equilibrium position when it floats in water of density ρ_W. It may seem strange, but for some values of ρ_B/ρ_W the buoy will only be stable floating vertically, and for other values it floats horizontally (any idea how this works?).
- Now suppose the buoy is pushed down so that it oscillates (bobs) up and down; assuming that the pressure is hydrostatic, find the equation of motion in terms of the vertical displacement $\eta(t)$, and find the frequency of oscillation.
- if the buoy is shaped like a cone instead of a cylinder, how does the vertical restoring force depend on η? In the language of classical mechanics, a 'hard spring' is one whose Hooke's law coefficient increases with increasing $|\eta|$ while a 'soft spring' is one where it decreases.