

## GFD Lab Demo 2 (2004) (#3 in 2001) Coriolis effects in a rotating fluid.

19 i 2001/22 i 2004

P.B.Rhines, E.G.Lindahl

### Summary

The demonstration of the observable nature of the Earth's rotation by Foucault is as famous as it is mysterious. The plane of oscillation of a pendulum swinging freely at the North Pole appears to rotate in 12 hours. If the pendulum is disconnected from the Earth, then it is just a matter of the observer's frame of reference; the pendulum's plane is fixed in an 'inertial' non-accelerating frame. However a bead moving freely on the surface of an 'ice-covered' Earth is a bit more interesting. The bead is subject to the gravitational pull of the planet and also to the acceleration implicit in its mean rotation. The geopotential,  $\Phi$  is the function whose gradient equals (minus) the inward gravity force plus outward centrifugal force (the *inertial* force due to the centripetal acceleration of particles toward the center of their circular orbits) force due to rotation:

$$\Phi = -\frac{GM}{r} - \frac{1}{2}\Omega^2 r_1^2$$

where  $r_1$  is the cylindrical radius from the axis, and  $r$  is the spherical radius from the planets center.  $G$  is the graviational constant, and  $M$  the mass of the planet; spherical symmetry is assumed with little error. The signs in the expression for  $\Phi$  give it a maximum at intermediate radius, decreasing to either side.

The surfaces of constant  $\Phi$ ...equipotential surfaces...are very interesting; near the Earth's surface they are nearly spherical, with a slight ellipsoidal bulge (the Equator having larger radius than either Pole, by about 21.4 km. At very large distance from the Earth they are cylinders parallel with its axis. At the transition, where  $\Phi$  is a maximum, geostationary satellites can sit over the Equator permanently...though unstably (they must be frequently corrected so as not to wander off the potential hilltop).

In the laboratory, with a rotating table, we have our own system of geopotential surfaces. Here gravity acts straight downward, and centrifugal acceleration is 'cylindrically' outward:

$$\Phi = gz - \frac{1}{2}\Omega^2 r_1^2$$

where  $z$  is the simple vertical coordinate and now  $r_1$  represents the radial coordinate in a polar coordinate system centered on the rotating platform. Equipotential surfaces,  $\Phi = \text{const.}$  are simple paraboloids.

### (i) Equilibrium rotation

A fluid spinning for a long time in a container on a rotating table will eventually rotate with the table, like a solid body; this is because of friction forces that will eat away at any relative motion other than this state.

Apparatus:

- rotating table
- clear-walled cylinder or large beaker, at least 30 cm. diameter
- ping-pong ball
- modelling clay
- scrap nuts and bolts
- small carpenter's level
- food color or blue ink
- eye-dropper

Level the table so that 'tides' will not occur. Center the cylinder or bowl on the table, fill it about  $\frac{1}{2}$  full and rotate at about  $2 \text{ rad. sec}^{-1}$  (3 sec. rotation period).  $33\frac{1}{3}$  r.p.m. is a fine speed for this experiment. As the rotation begins notice the flat free surface of the water begin to slope at the rim, gradually filling out the paraboloid characteristic of an equipotential surface. Check quantitatively, with a scale, that the above formula is correct. Why is the free surface an equipotential? Because the air is so much less dense than water (by a factor  $1/800$ ) that the pressure at the free surface is nearly the same everywhere. Thus the pressure gradient is perpendicular to the surface, and the 'hydrostatic' momentum balance equates this gradient to  $\rho \nabla \Phi$ ; hence  $\Phi = \text{const.}$   $p = \text{const.}$  and  $\rho = \text{const.}$  surfaces all are the same paraboloids for solid-body rotation.

*Floating.* Archimedes' principle supposes that a submerged object experiences the same pressure forces from the surrounding water as would the water that is 'displaced' by the object. This gives a unified idea of buoyancy, in which fluid parcels feel it just as would an immersed solid. What about a floating object? The argument applies to the submerged part, but the above-water part of the object led us to consider stability and roll-over. Here it is a more serious problem. Place a ping-pong ball on the free surface, near the edge. If it were entirely equivalent to the water it displaces, it should stay there, but it does not. The center of mass of the ball is closer to the rotation axis than was the center of mass of the displaced fluid (make a sketch showing the resultant of the gravity plus centrifugal forces for the ball and displaced water). The outward centrifugal force is thus weaker, and hence the ball skates down toward the

center. But if its density were to approach that of water, it would be ever closer to being in equilibrium anywhere on the paraboloid.

To show that things don't really want to fall 'down the hill', create a ball with a 'keel' by sticking a bit of clay with a metal nut or two inside it, to the ball. Drop this near the center of the free surface, and you should see it skate outward, 'up the hill', because its center of mass is *below* that of the displaced eyebrow of water.

*Sinking.* Since we have new horizons and a new 'vertical', things should sink perpendicular to the paraboloid rather than down the z-axis. To see this, gently put some dense dye on the surface (or, fine solid dye powders like methylene blue). As they begin to descend they move outward, forming a 'beard'.

*Taylor-Proudman effect.* Now we want to establish one of the key properties of a rotating fluid, the *stiffness* induced by rotation. Dilute some food coloring or blue ink with water (so that it is still nearly opaque, but not so strong). It is best to have a white background beneath the cylinder. Drip a few drops onto the free surface and give the fluid a quick, gentle, brief stir with a flat stick, like a ruler. The dye will deform in a complex fashion, but you will begin to see that it forms vertical sheets or 'curtains' rather than becoming totally mixed up. Viewed from above the dye may still be in very thin tendrils, as a result. For comparison do the same thing to a bowl of water that is not rotating, and compare.

The rotation of a constant density fluid, in effect, threads it with *vortex lines* lying parallel to the rotation axis, these lines are 'elastic', and if deformed they, through Coriolis forces, will try to restore their vertical orientation. If the motions you create by stirring are not too violent, this vertical rigidity is overwhelmingly strong.

*Inertial waves and oscillations.* This demonstration requires more sophisticated apparatus, but is worth creating if resources allow. A paddle driven by an electric motor makes a regular, periodic disturbance in the fluid, which is imaged with a video camera rotating with the table, and recorder with a time-lapse video recorder (or, a still camera makes a time-exposure photo of bright particles in the fluid).

The result is a remarkable picture of elasticity given the fluid by rotation. By disturbing the fluid more rapidly than in the Taylor-Proudman flows above, we bend the vortex lines periodically. Fluid particles feel both pressure gradient and Coriolis force; they move approximately in circular orbits, while planes of fluid move in 'sheets'. The natural frequency of this system is  $2\Omega$ , which is known as the Coriolis frequency. Thus, a free particle at the North Pole of an ice covered world can

experience a circular oscillation with a period of 12 hours. Elsewhere on Earth it is the vertical component of the rotation vector that provides a Coriolis force, and so the frequency is in general  $f \equiv 2\Omega \sin(\text{latitude})$ . In the full, 3-dimensional inertial waves, fluid is moving at an angle, say  $\alpha$ , to the horizontal, which is determined by the wave frequency imposed by the paddle, say  $\sigma$ . In fact,  $\sigma = 2\Omega \sin \alpha$ .

A demonstration of a basic inertial oscillation can be constructed. For example, it might be an interesting project to put the rotating apparatus in a freezer until the water solidifies with a paraboloidal surface, rather as one does to produce a fine telescope mirror (with wax that solidifies). Then, a marble can be rolled on this 'roulette wheel'; viewed with video that rotates with the table, the inertial oscillation is striking. But it is also easily imagined, and perhaps not worth the trouble to reproduce. It is easier simply to mount a pendulum on the rotating table and view it with the rotating camera. The key parameter is the ratio of pendulum period to rotation period,  $(g/L\Omega^2)^{1/2}$ ; above a threshold rotation, the pendulum has two possible rest positions, one vertical and the other at a finite angle to the vertical. There is no magic in this: the pendulum bob is moving in a circle, relative to inertial space, and can do so steadily at an angle  $\theta = \text{inverse cos}(g/\Omega^2 L)$ . The pendulum can swing about this angle in something resembling inertial oscillations.

The game of Coriolis Catch is worth playing. If a rotating playground carousel can be found, take a tennis ball and a friend, and attempt a game of catch while rotating. The circular path followed by a thrown ball is very striking to see. Be careful not to yield to centrifugal forces or sea-sickness. Doctors tell us that the inner ear is very sensitive to acceleration, as it works with pressure imbalance in fluid filled tubes...a sort of biological manometer. A uniform acceleration is o.k., since it simply acts a reorientation of gravity. But, moving your head too much can create sudden *mal-de-mer*.

### (ii) Coriolis force on a jet

The film, 'Rotating flows' by David Fultz, a part of the classic Fluid Mechanics Film Series (available from Britannica Films, Chicago, Ill.) contains wonderful scenes of inertial waves and deflected smoke-rings on a rotating table. Here we make a simpler demonstration of Coriolis.

#### Apparatus

- rotating table
- cylinder (as large in diameter as possible)
- small (~200 ml) Beaker and with capillary tube outlet

food color

If the capillary tube is not available, substitute an eye dropper, connected to the beaker through a siphon. Arrange the beaker so that it can be affixed to the cylinder, with the outlet directed horizontally across the fluid (submerged a few cm. beneath the surface). Fill the cylinder and bring it up to speed. Use a very low rotation rate if possible. If the cylinder is large (50 cm or larger in diameter) set  $\Omega$  as small as  $0.05 \text{ radian sec}^{-1}$ .

Start the colored jet flowing horizontally across the fluid. It will bend to the right due to Coriolis, forming an 'inertial circle' with radius about  $U/f$ , where  $U$  is the typical velocity of the jet. If a capillary tube and very thin jet is present, the experiment is particularly beautiful. A mushroom shaped vortex forms at the initial injection, and moves at the front of the jet. Eventually the jet may cease to advance, and wind up, feeding the vortex (which has by now been morphed from the ring vortex at the beginning into a single vortex with vertical axis). The thinness of the jet in this gently forced case is somewhat of an illusion. Mark nearby fluid with dye of another color, perhaps in straight lines. These will show that the jet entrains much more fluid from the sides, and drives a broad flow that is invisible without these other markers.

Taylor-Proudman effects are at work, and the flow grows in vertical extent. It is remarkable how voluminous a flow can be driven by this tiny forcing effect.

### **(iii) A tornado vortex**

A dramatically different form of rotating fluid is formed when fluid is withdrawn from the center of a cylinder, as in a 'bathtub vortex'. When this is balanced by injection of fluid at the rim, with some angular momentum, the vortex can be stabilized and viewed for as long as desired.

#### Apparatus

tall clear-sided cylinder, at least 30 cm diameter, with centered hole in  
base  
sink with tap fit to Tygon tubing  
ping pong balls  
dye

Organize the cylinder on a platform in a sink so that the inflow tube can be clamped to its rim, and the outflow goes down the drain. A stopper that fits the bottom hole is

useful. Fill the cylinder and remove the stopper, keeping the inflow going so as to maintain the free surface height.

The free surface dances as it dips into a parabolic shape, and given sufficient angular rotation, the air column descends to the drain. The spin of the vortex is very strong, being essentially the conserved angular momentum inherited from the outer region of fluid. Drop dye into the flow and you will see cylindrical surfaces that last a very long time. How can this be? The water does not seem to flow simply inward to the center. In fact, in a very tall cylinder it will do so, but the rigid bottom of the cylinder tends to be a site of radial inflow, so that the dye cylinders may be bypassed by flow beneath. Drop small particles and permanganate dye crystals into the flow to try to find this pathway. A form of inertial oscillation can be seen on the vortex column: it has elastic properties.

A ping-pong ball dropped into the flow may find a stable point where downward suction balances upward buoyancy. Dye then released near it takes on a remarkable pattern.

The shape of the free surface is of great interest. Being a surface of nearly constant pressure, we can use Bernoulli's equation and angular momentum conservation to estimate that the azimuthal velocity varies like  $1/r$ , and hence the free surface height varies like  $1/r^2$ . This is a striking contrast to the  $r^2$  free surface shape with solid-body rotation.

Oscillations of the vortex column are easily stimulated, and are related to inertial waves. Surface waves and ripples are carried down the tube giving it a complex texture.

At this point, we might enter into a discussion of actual tornadoes, their similarities and dissimilarities to this sink-vortex. Concentration of angular momentum is common to both, and provides the enormous winds and low pressure in atmospheric tornadoes. But there is much more science beyond this: why are most tornadoes cyclonic? Why do more than 90% of the tornadoes on Earth occur in the central plains of the US? [Dr. Douglas Lilly tells me that dry air at altitude, coming off the high Mexican plateau, caps off the moist air from the Gulf of Mexico, somehow intensifying the super-cell storms.] How does the boundary layer interact with inflowing air?

The power of such a vortex may have been exaggerated in films like Stephen Spielberg's *'Twister'* (1995), which despite consulting many of our atmospheric

sciences colleagues, used entirely digital imitations of tornadoes rather than (much scarier) real film footage. But not by much: a film of a recent tornado in Texas showed four pickup trucks orbiting together high overhead!

#### **(iv) Rotating flow over a mountain; the Rossby number**

We now look at a flow with real application to oceans and atmospheres. The resistance of a rotating fluid to vertical stretching, when the Rossby numbers are small, leads to a wonderful, nearly paradoxical, result that G.I. Taylor developed in 1913. If a small sphere is pushed across a rotating fluid, normal to the rotation axis, fluid must move aside as it passes; in a frame of reference moving with the sphere, a steady flow might develop, roughly as in a non-rotating fluid. Above and below the sphere there would seem no reason for the fluid to be deflected and, indeed, idealized potential flow of a non-rotating fluid these deflections die off like  $1/r^2$ . But, here, the vertical stiffness of the fluid dictates that fluid above, below and at the level of the sphere will be slaved to move together. It is as if a virtual cylinder (circumscribing the sphere) extends throughout the fluid. A corollary is that very little fluid should deflect vertically to flow up-and-over the sphere; it must go around, staying nearly in a single horizontal plane.

#### Apparatus

- rotating table
- clear cylinder, as large diameter as possible
- modelling clay
- dyes
- fish flakes

This experiment is greatly aided by an on-board video system and time-lapse video recorder, but it is still very interesting in its simplest form.

Fashion a clay mountain, typically 15% as wide as the cylinder, and stick it to the floor of the cylinder, at about  $2/3$  the radial distance from the center to the rim. Fill the cylinder to a depth of at least 20 cm, apparatus permitting. Spin up the system to a solid body state. The best rotation rate is between  $0.5$  and  $1. \text{ rad sec}^{-1}$ , though the record player speed of  $2 \text{ rad sec}^{-1}$  is acceptable. If a record player is being used, there must be some way to change its speed; a variac a.c. voltage control can do this to some degree (see hardware chapter).

Place some dots of dye near the mountain and increase slightly (about 5%) the rotation rate. The fluid above the mountain, when it is pushed off, is vertically

stretched as it moves into deeper water. This stretches the planetary vortex lines that thread vertically through it. Positive relative vorticity develops: a cyclonic eddy. Conversely, fluid forced up over the mountain is squeezed vertically and becomes shorter. An anticyclonic eddy appears, which is bound to the mountain. In simplest terms, the stretched fluid conserves its angular momentum (measured in the frame of reference of the room) and like the figure skater with arms drawn inward, spins faster.

The dye will show how well the columnar nature of the flow is enforced by rotation. But there is a slight disagreement with G.I. Taylor, who emphasized the avoidance of the obstacle which should act like a tall, solid cylinder due to the stiffness effect. What we discover here is that fluid does penetrate the space above the mountain, but in doing so suffers a strong vorticity penalty. Scale analysis shows that significant deflection by the mountain occurs when  $\delta h/h > U/fL$ , the Rossby number. The mountain height is  $\delta h$ , full depth is  $h$  and its radius is  $L$ .  $U$  is the fluid horizontal velocity in the rotating frame of reference.

The experiment can be repeated many times; an interesting way to do it is to increase the rotation rate for just a few seconds, then bring it back to its original value. This will ‘fling’ a column of fluid off the mountain, but leave it nearby to develop. The distance it is flung is an important parameter. If small, the two vortices, with positive and negative vorticity will interact as a dipole pair. But one is bound to the mountain, and so the other rotates around it. This is actually a form of Rossby wave motion which will arise in a later lab. When the distance separating the vortices is greater, the free cyclone will rotate slowly round the mountain, dying off rather quickly. Vortex interactions of this kind are an important research area in GFD, relating in obvious ways to atmospheric and oceanic flows.

Time-lapse videos, taken from above, reveal much more of the eddy structure. In accelerated time you will see the vortex pair ‘violently’ interacting, and then quickly dying due to friction. The time for viscosity to spin-down the fluid motion is surprisingly brief, of order 250 sec. for  $f = 1$ . This is the result of special rotating boundary layers that transmit viscous effects efficiently to the interior.

While this apparatus is set up, it is interesting to change the water and add fish flakes to visualize the shearing motion in the fluid. If the lighting is appropriate, you will see the ‘Taylor column’ above the mountain outline with a silvery surface, as the flakes orient in the shear.

*When is rotation important?* In this series of experiments we frequently ask ‘when is rotation important’. The simplest answer is that, when the time-scale of a fluid

event is of order  $f^{-1}$  or longer, then Coriolis effects should be considered. Often the fluid-event time-scale is just  $L/U$ , the ratio of horizontal length scale to horizontal velocity. The ratio is then

$$Ro \equiv U/fL$$

If, however there is another intrinsic time-scale  $T$ , for example the period of a wave, then a second Rossby number

$$\varepsilon \equiv 1/fT$$

measures Coriolis effects. When friction is important the natural fluid time-scale can be simply  $L^2/\nu$  or  $H^2/\nu$ ,  $\nu$  being the kinematic viscosity. In this case another measure of the importance of Coriolis effects is the Ekman number,

$$E = \nu/fL^2 \text{ or } \nu/fH^2$$

(corresponding to horizontal and vertical shear: the latter is usually dominant in nature). Does the pond in your back yard feel the rotation of the Earth? Very possibly, if there are flows in it with time-scale greater than about 4 hours. To decide you might try to make a time-lapse video of the surface of the pond, which will be responding to wind, temperature change and inflow/outflow. At quiet times, slow circulations of the pond may indeed be pressed to the 'right-hand' shore. Larger lakes have clearly document Coriolis response, visible in the tilted isothermal surfaces within currents, in slow wave motions, and in boundary layer flows (all of these to be discussed in subsequent lab chapters).