Geophysical Fluid Dynamics I   P.B. Rhines

Problem Set 2

out: 22 Jan 2004

back: 29 Jan 2004 *note change in formula for velocity field in problem 1, and sign of ψ in problem 4*

1. Geostrophic flow. A vortex with circular streamlines can have velocity field of many different kinds; the tornado vortex in the lab has approximately u_θ = A/r azimuthal velocity ('swirl' velocity) which has uniform angular momentum. But this goes to ∞ at r=0, so a good model of such a vortex is

$$\begin{aligned} u_\theta &= U_0 \frac{r}{a} \quad r < a \\ &= U_0 \frac{a}{r} \quad r \ge a \end{aligned}.$$

Sketch the velocity as a function of r.

Solve for and plot the pressure field that balances this velocity

(a) without Coriolis effects (Ω=0)

(b) with Coriolis effects ($\Omega \neq 0$)

Discuss the momentum balance in the radial direction, showing how it depends on the Rossby number, which here can be defined as Ro = $U_0/2\Omega a$. You will need to write the momentum equation in polar coordinates (or see Batchelor's text or others), dropping the time-varying terms (∂/∂t…) and frictional terms. Consider both cyclonic and anticyclonic vortices (U_0 postive and negative, respectively). Describe how the correspondence between 'cyclonic' and low pressure; 'anticyclonic' and high pressure depends on Ro.

2. Write a one-page essay on one of the lab experiments you have seen, taking it as far as you can: stating the experimental situation, the question in mind, what you observed, and what the implications to the atmosphere or ocean may be.

3. Large scale flows, if slowly varying in time, tend to be in geostrophic balance,

$$2\vec{\Omega} \times \vec{u} = -\nabla p / \rho + O(Ro) + O(1/\Omega T)$$

in the horizontal and hydrostatic

$$\frac{\partial p}{\partial z} = -\rho g$$

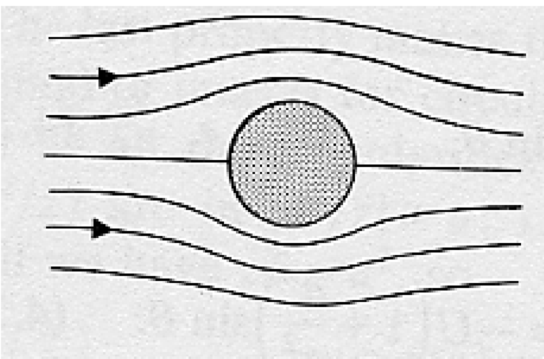
in the vertical. Consider the Antarctic Circumpolar Current (ACC) which travels eastward round the Southern Ocean. It has an average volume transport of roughly 140 x 10^6 m^3 sec^{-1}. (140 Sverdrups), and is largely wind-driven, by the westerly winds overhead.

Suppose the ACC is 1000 km wide, and the eastward velocity is uniformly distributed over this width and over a depth range of 2000m.

- Find the shape of the sea surface for the case of hydrostatic, geostrophic balance.
- Imagine the wind starting to blow round this belt (centered at 60^0 S latitude) with the ocean at rest. How would you expect the sea surface to change from its initially flat state? Ignore the presence of continents (this is an 'aqua-planet).

4. Consider a flow around a cylinder: this is a classic fluid dynamics problem. If the fluid has zero vorticity ('irrotational flow'), the solution for the streamfunction, ψ, is

$$\psi = -U \sin\theta (r - a^2 / r)$$



which is the sum of a uniform flow in the x-direction plus a dipole source-sink distribution. See, e.g. Acheson, Elementary Fluid Dynamics, sec 4.5. Use Bernoulli's equation to find the pressure field, assuming the flow far upstream has uniform pressure. This part can be found in any basic fluid dynamics text.

Now add rotation: as shown in class, the velocity field can still be the same, if Coriolis forces are balanced by an added pressure field. Solve for this new pressure field, by assuming that far upstream the flow is geostrophic with $p = \rho f \psi$ for $x \Rightarrow -\infty$ (f is the Coriolis frequency, $2\Omega \sin \varphi$). Sketch the result, showing the transition from a strongly rotating to a weakly rotating flow, as a function of the Rossby number which here can be defined sensibly as U/fa.

The procedure here is to find the new component of pressure (due to rotation) at a point r,θ by finding the value of ψ at that point, and back-tracking to x=-∞ where that value of ψ corresponds to a known value of p.