

## GFD-1 Winter 2004 Take-home Quiz #1

out: Thursday 5 February

back: Friday 6 February in class (or before 3.00pm to Dan in At.Sci. 620 or  
PBR in Ocean Sciences 319)

Books and notes may be used, but please do not communicate with anyone about this quiz.

### 1. (short answers)

a. If you are holding a birthday balloon in an elevator which rises quickly from the ground to the 100<sup>th</sup> floor of the GFD Building, describe how the tension in the string varies with time.

b. Why do we say the surface elevation of water is a 'pressure gauge' for the flow below.

c. Which of the following flows is closer to being in geostrophic balance:

(i) An atmospheric cyclone, of diameter 1000 km and peak wind-speed of  
20 m sec<sup>-1</sup>;

(ii) an oceanic circumpolar current with currents of 0.5 m sec<sup>-1</sup> and width 100 km.

2. The geopotential surfaces,  $\Phi = \text{constant}$ , are 'horizons' along which a particle can move without doing work against the sum of gravity and centrifugal forces. An idealized form of  $\Phi$  is

$$\Phi = -GM/r - \frac{1}{2} \Omega^2 r^2 \cos^2 \varphi$$

where  $r$  is the spherical radius and  $\varphi$  is the latitude. The other parameters are given at the end of the quiz..

Make a sketch of  $\Phi(r)$  at the Equator ( $\varphi=0$ ) and pole ( $\varphi = \pi/2$ ) and indicate the difference in radius of a constant- $\Phi$  surface between Equator and Pole. What parameter describes the difference in radius of a geopotential surface at the poles and at the Equator? (Hint: think about  $[\Phi(\text{Equator})-\Phi(\text{Pole})]/\partial\Phi/\partial r$ .)

3. Consider a 2-dimensional, single-layer, constant-density fluid with uniform rotation,  $\Omega$ . This is a variation of the 1-dimensional geostrophic adjustment problems done in class. For the two-dimensional initial condition

$$\eta = \eta_0 = A \cos k_0 x \cos l_0 y; \quad u=0; \quad v=0 \quad \text{at } t=0$$

find the solution for  $\eta(x,y,t)$ : both the time-average and time-dependent parts, and draw sketches of the particular solution and interpret it as fully as you can. There are no side boundaries, and the mean depth is a constant ( $H$ ). The appropriate linear equations are given at the end of the page. With variation in  $x,y$  and  $t$  this partial differential equation set can be solved by separation of variables, which involves trying a form of solution that fits the initial condition  $\eta_0$ . Briefly, what would you expect to happen if we added a side-wall (a 'coast' or Equator) where the normal component of horizontal velocity equaled zero?

4. Energy flux and tidal dissipation in a bay. We have seen several kinds of 'conservation-flux' equation,

$$\frac{\partial C}{\partial t} = -\nabla \cdot \vec{F}$$

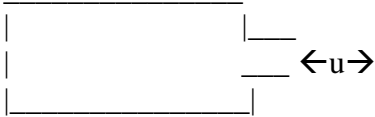
in which the concentration  $C$  of some property of the fluid (e.g., kg salt per kg of seawater) changes in time due to a flux  $\vec{F}$  of that property into the region. The 'advective' flux often has the form  $\rho \vec{u} C$  where  $\rho$  is density (kg m<sup>-3</sup>) and  $\vec{u}$  is velocity.

For energy it is a little different because energy can be *transmitted* by pressure forces. In fact, the linearized wave equations omit the advective flux altogether, and just retain the transmission of energy (see Gill sections 4.6 and 5.7).

Suppose that we have measurements of the sea-surface height  $\eta$  and horizontal velocity  $u$  at the entrance to a large bay. These are due to oscillating tides (a long gravity wave) observed to be of the form

$$\eta = A \cos(\omega t + \phi_1), \quad u = B \cos(\omega t + \phi_2)$$

where  $\phi_1, \phi_2$  are constants. Calculate the time-averaged energy flux into the bay, assuming hydrostatic, long wave dynamics. Presumably, if this measured flux is non-zero, the energy is going into frictional dissipation within the bay. The mean depth is  $H$ , and width of the mouth of the bay is  $L$ .



The mean radius of the Earth is about 6370 km. The gravitational constant  $G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ ,  $M = 6 \times 10^{24} \text{ kg}$  is the mass of the Earth,  $\Omega = 0.727 \times 10^{-4} \text{ sec}^{-1}$

The linearized equations for a single layer of constant density fluid with a free surface are:

$$u_t - fv = -g\eta_x$$

$$v_t + fu = -g\eta_y$$

$$\eta_t = -H(u_x + v_y)$$

where  $H$  is the constant mean depth,  $u, v$  are horizontal velocity components and  $\eta$  is free-surface elevation.