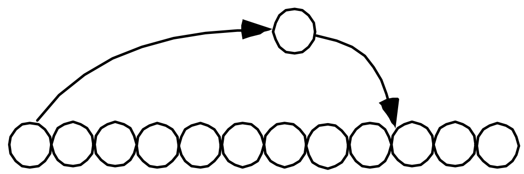
Bedload

When $u_*/w_s << 1$, but $\tau_0 > \tau_c$, the dominant mode of transport will be saltation.



Saltation is an important mode of transport for sandy and gravelly systems.

There have been an extremely large number of efforts to assess bedload transport. Gilbert, Bagnold, Einstein, Engelund, Vanoni, Shields and Straub all have developed theory and formulae.

Many of the 'recent' formulae take the form

$$q_b^* = a(\tau_* - \tau_{*c})^b \tag{1}$$

where q_b^* is the dimensionless bedload sediment discharge per unit width

$$q_b^* = \frac{q_b}{\sqrt{RgD^3}} \tag{2}$$

and a and b are empirical constants. D is the particle size in mm. q_b in this expression is a volumetric flux and therefore has units of L^2/T.

These models rely on a 'sheet-flow' assumption. The most popular of this type of model is Meyer-Peter and Müller

$$q_b^* = 8(\tau_* - \tau_{*c})^{1.5} \qquad (3)$$

It works well for wide channels, not in flood. It is also helpful if the bed material is well-sorted sand (i.e., capacity-limited streams).

Einstein (1950) proposed a more general formulation. It relies on a stochastic approach to transport of saltating particles.

He made several assumptions upon which the model was built. They are (Graf, 1971):

- A steady and intensive exchange of particles is assumed to exist between moving bedload and the bed.
- The bedload moves slowly downstream with respect to the fluid: A.k.a. **Exner approximation.**
- The individual jumps are large with a long hiatus between jumps.
- The average step made by any bedload particle appears to be independent of the flow condition, the transport rate, and the bed composition.

- Different transport rates can be achieved by a change of the average time between two steps and of the thickness of the moving layer.

The deposition rate will be related to

$$\frac{q_{bw} i_s}{(A_L D)(\rho_s g k_2 D^3)} = \frac{q_{bw} i_s}{(A_L \rho_s g k_2 D^4)} \qquad (4)$$

where $A_L D$ is the length over which an individual particle travels before it deposits, D is the particle diameter, q_{bw} is the weight flux of sediment per unit width, i_s is the fraction of bedload in the ith size, and k_2 is a shape factor.

Erosion, or removal, of sediment from the bed will be a stochastic process. Namely, it will simply be the probability of an ejection multiplied by the material to be ejected.

$$\text{Erosion} = \frac{i_b p}{k_1 D^2 t_e} \qquad (5)$$

where p/t_e is the probability of an ejection, i_b is fraction of bed material of a particular size, is the grain diameter and k_1 is another shape factor.

The exchange time t_e is required because the exchange will not be instantaneous. It can be approximated by

$$t_e \sim \frac{D}{w_s} = k_3 \sqrt{\frac{D}{Rg}} \tag{6}$$

where k_3 is a dimensionless time constant.

If we are at equilibrium, then (4) and (5) will be equal –

$$\frac{q_{bw} i_s}{\left(A_L \rho_s g k_2 D^4\right)} = \frac{i_b p}{k_1 k_3 D^2} \sqrt{\frac{Rg}{D}} \tag{7}$$

There are two unknown quantities in (7). They are the dimensionless distance a particle travels before it is deposited A_L and the probability of movement p. We want to calculate A_L in terms of the fixed, constant jump length λ_b –

$$A_L D = \sum_{n=0}^{\infty} (1-p) p^n (n+1) \lambda_b D = \frac{\lambda_b D}{1-p} \tag{8}$$

Rearranging (7) with the knowledge of (8), we find

$$\frac{p}{1-p} = \left(\frac{k_1 k_3}{k_2 \lambda_b}\right)\left(\frac{i_s}{i_b}\right)\left(\frac{q_{bw}}{\rho_s g} \sqrt{\frac{1}{RgD^3}}\right) \tag{9}$$

Because the LHS is reflective of the amount of material leaving the bed at one time, Einstein (1942) referred to variable quantity on the RHS, as the intensity of bedload transport Φ.

$$\Phi = \frac{q_{bw}}{\rho_s g}\sqrt{\frac{1}{RgD^3}} \tag{10}$$

We can then write (9) in terms of this new variable and condense the constants to form

$$\frac{p}{1-p} = A\Phi \tag{11}$$

where *A* is an empirical constant.

However, we still need to relate the probability of sediment movement to flow parameters. To do this, Einstein postulated that the probability *p* was related to the bulk quantities –

$$p = f\left(\frac{k_2 RgD^3}{0.5 c_L k_1 D^2 {u_b}^2}\right) \tag{12}$$

We assume that the velocity of interest u_b is the velocity at the edge of the laminar sublayer –

$$u_b = 11.6 u_* = 11.6\sqrt{gHS} \quad (13)$$

where H is the flow depth (R_h in Einstein's original theory). Regardless, substituting (13) into (12)

$$p = f\left(\frac{k_2 RgD^3}{62.5 c_L k_1 D^2 gHS}\right) \quad (14)$$

which leaves the variable portion to be defined as the **flow intensity** Ψ

$$\Psi = \frac{RD}{HS} \quad (15)$$

which can be substituted back into (14) to form

$$p = f(B\Psi) \quad (16)$$

where B is another empirical constant. Combining (16) and (11), we find

$$A\Phi = f(B\Psi) \quad (17)$$

Initially, Einstein (1942) left the relationship of Φ and Ψ as a purely empirical one. However, he later (in 1950) set forth an analytical argument for the relationship between these two variables.

The results of this along with the original data of Gilbert (1914) and Meyer-Peter and Müller are shown below.

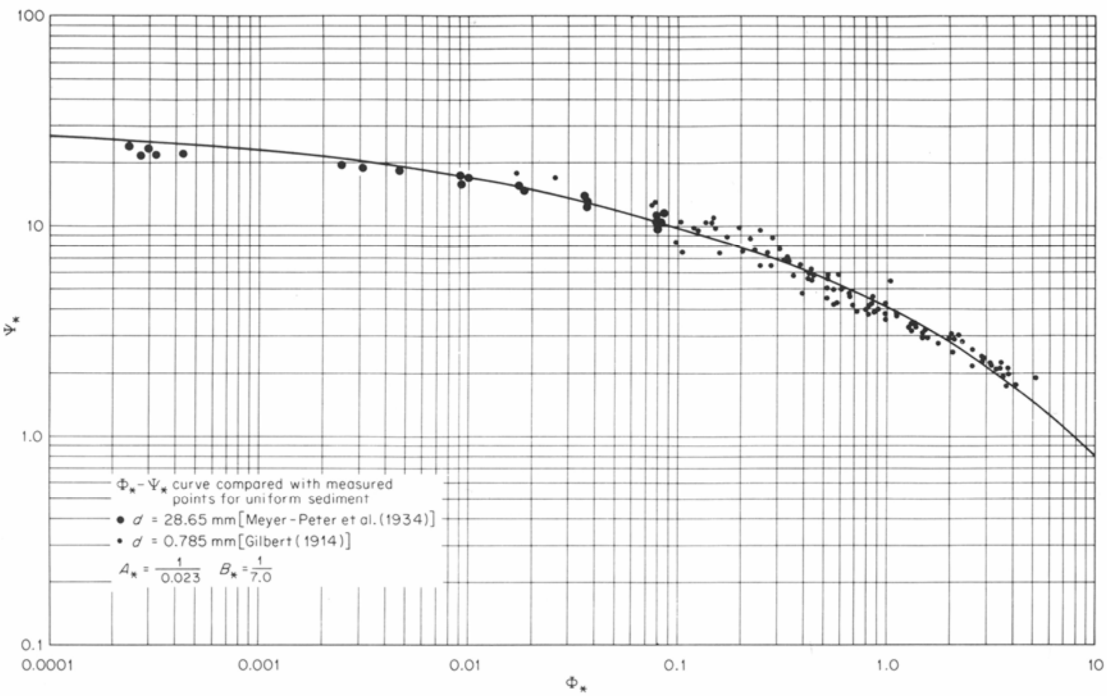


Fig. 7.13 Plot of Einstein's functions; Φ_* vs. Ψ_*. [*After* Einstein (*1950*).]