Open channel flow

Open-channel flows are a special class of boundary-layer flows that are confined to a channel form.

Why are open-channel flows important?

- Many natural systems responsible for the transport of sediment are channelized, in both subaerial and subaqueous environments.
- Nearly all of the modeling performed on the entrainment and transport of sediment is either in open channels or in 1-D boundary layers.

Like any fluid mechanical problem, dimensional analysis can play a key role.

In open channel flow, there are only a few variables that are needed to describe transport. They are…

U = velocity [L/T]
L = relevant length scale [L]
μ = dynamic viscosity [M/(LT)]
ρ = density [M/L^3]
g = gravitational acceleration [L/T^2]

Different velocities (u_*, u_{max}, v, etc.) and length scales may be used (R, H, W, P). Typically if you are studying a particular set of physics, only one or two of these will be important (e.g., sediment transport is highly dependent on u_*)

Buckingham's Pi thereom states that there are *n-r* dimensionless Π coefficients, where *n* is the number of independent physical quantities and *r* is the number of dimensional units (i.e., M, L, T) that describe a physical system.

There are many ways to generate these two dimensionless quantities (which can generate many different forms). However, the most conventional terms are

$$Fr = \frac{U}{\sqrt{gL}} \tag{1}$$

$$Re = \frac{\rho UL}{\mu} = \frac{UL}{\nu} \tag{2}$$

Therefore, if these two dimensionless parameters can be matched, the physics of the flows should be 'similar'. The difficulty is that water has a relatively low viscosity. For instance, in a 1:100 length scale model, the viscosity has to be:

$$\nu_{model} = 10^{-3} \nu_{natural} \quad (3)$$

Alternative… Reynolds similarity

The Froude number can be thought of as:

- the ratio of the wave speed to mean speed of the flow

- the ratio of potential energy to kinetic energy (or the ratio of buoyancy to inertial forces)

- a description of the minimum energy state of the flow

Flow regime and Froude number effects

The Froude number dictates the 'flow regime'. Principally, there are three regimes of flow: supercritical ($Fr > 1$), subcritical ($Fr < 1$) and critical ($Fr = 1$).

Ramifications? Hydraulic control – location within flow that regulates flow rate.

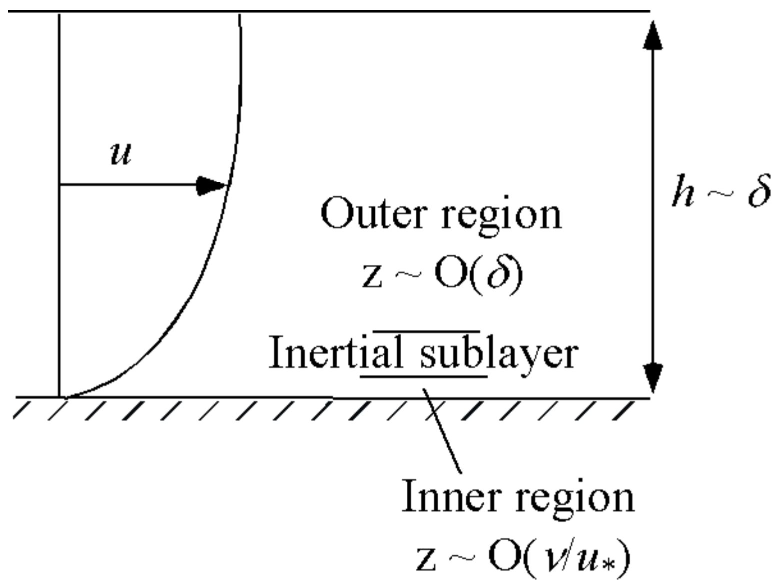
Supercritical flow – Hydraulically controlled from upstream.

Subcritical flow – Downstream control.

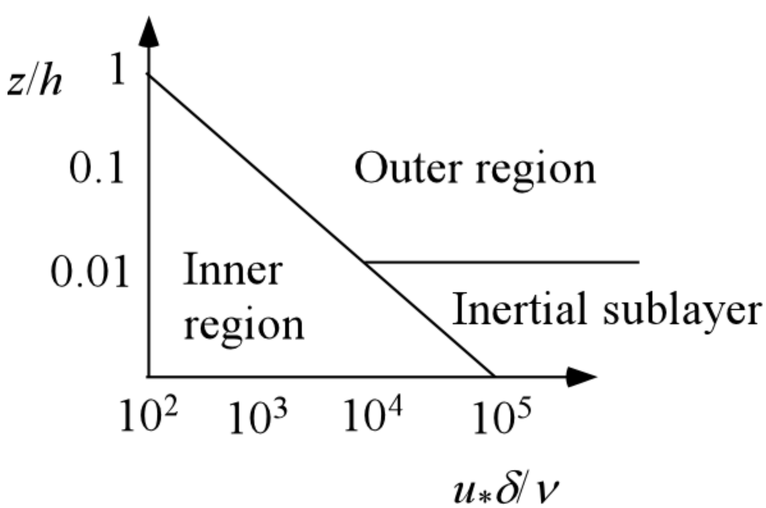
Critical flows do not exist naturally (they are unstable). If a flow passes through $Fr = 1$, a hydraulic jump will occur.

Velocity structure in wide, rectangular channels (i.e., boundary layers)

Removing dependence in the spanwise direction, assuming a smooth boundary and fully developed flow, the vertical velocity profile (now y) looks like:



Inner region, a.k.a., wall layer
Inertial sublayer, a.k.a. matching layer, intermediate layer



As we showed a few days ago, the streamwise (*x*) equation of motion (in general) for a boundary layer is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (4)$$

For Reynolds-averaged, steady flows with negligible acceleration (i.e., relatively constant forcing)

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{d\left(\overline{u'v'}\right)}{dy} + \nu\frac{d^2\overline{u}}{dy^2} = 0 \qquad (5)$$

In the vertical (*y*):

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{d\left(\overline{v'^2}\right)}{dy} = 0 \qquad (6)$$

Integrating (6) along *y*

$$\frac{p}{\rho} + \overline{v'^2} = f(x) \qquad (7)$$

Because $\overline{v'^2}$ is independent of *x*, we find that the pressure gradient can be written

$$\frac{\partial p}{\partial x} = \frac{dp_0}{dx} \qquad (8)$$

where p_0 is the externally imposed streamwise pressure gradient, which is NOT necessarily hydrostatic.

Integrating (5) along y and using (8), we find

$$-\frac{y}{\rho}\frac{dp_0}{dx} - \left(\overline{u'v'}\right) + \nu \frac{d\bar{u}}{dy} = \text{Constant} \qquad (9)$$

Evaluating at $y = 0$, we define a shear velocity u_*

$$\nu \frac{d\bar{u}}{dy}\bigg|_{y=0} = u_*^{\,2} = \frac{\tau_o}{\rho} \qquad (10)$$

Assuming that no shear is generated at the upper interface

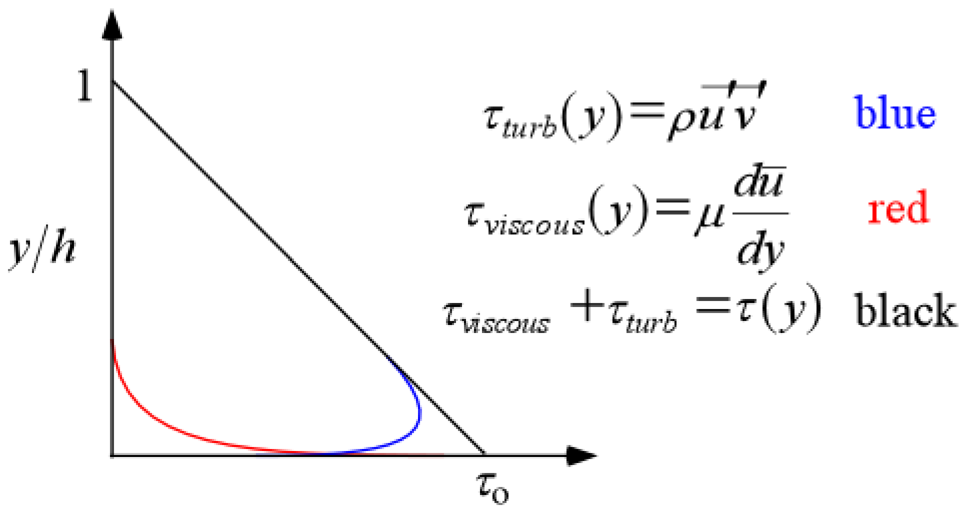
At $y = h$ => $$-\frac{h}{\rho}\frac{dp_0}{dx} - u_*^{\,2} = 0 \qquad (11)$$

which leads to the common assumption (for gravity-driven flows on small slopes) that

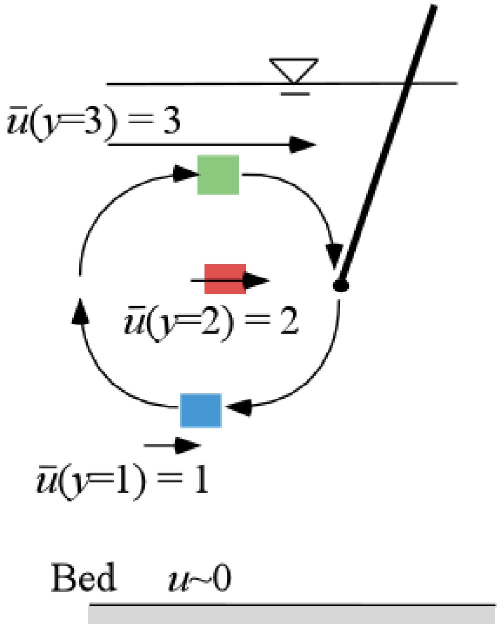
$$u_* = \sqrt{ghS} \tag{12}$$

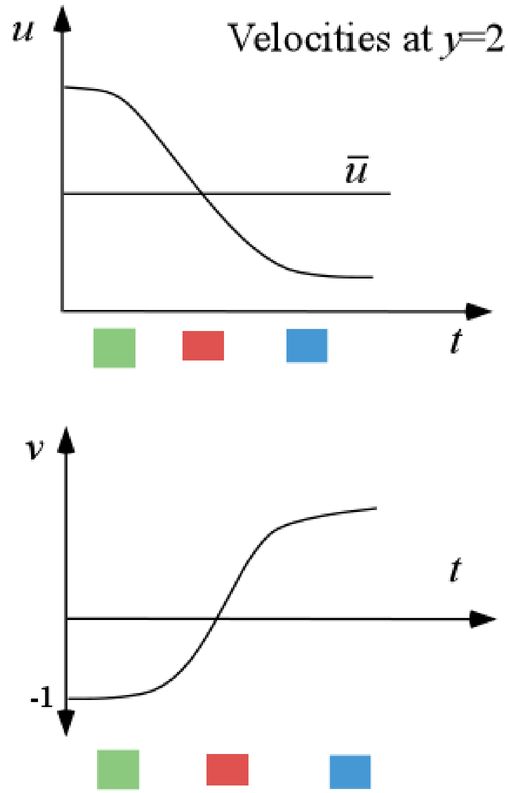
Simultaneous solution of (9) and (11) yields the vertical structure of shear

$$-\rho\left(\overline{u'v'}\right) + \mu \frac{d\bar{u}}{dy} = \tau_o\left(1 - \frac{y}{h}\right) \tag{13}$$



Reynolds stress $\overline{u'v'}$, what is it physically?





$$\overline{u'v'} = \frac{-1(1) + 0(0) + 1(-1)}{3} = -2/3$$

$\overline{u'v'}$ can only be important far from the wall (where viscosity is negligible), while near the wall ν dominates.

Two types of events can produce $\overline{u'v'}$ near the bed. They are called sweeps (Q4) and ejections (Q2).

Outer Region

In outer region, length scale = *h* and velocity scale = u_*

Nondimensionalizing (13) using these variables

$$-\frac{\left(\overline{u'v'}\right)}{{u_*}^2}+\frac{\nu}{u_* h}\frac{d\left(\bar{u}/u_*\right)}{d\left(y/h\right)}=1-\frac{y}{h} \tag{14}$$

Defining $Re = u_* h/\nu$, and assuming fully turbulent behavior (i.e., $Re \to \infty$), we find the equations of motion simplify to

$$-\frac{\overline{u'v'}}{{u_*}^2}=1-\eta \tag{15}$$

With some assumptions about the Reynolds stress and the distribution and the transport of turbulent energy –

$$-\frac{\bar{u}\big|_{y=h}-\bar{u}}{u_*}=F\left(\eta\right) \tag{16}$$

This formulation is known as the **Velocity-Defect Law**

In practice (e.g., oceanic currents, etc.), the quantity h can be replaced by the height at which the second derivative switches sign. In rivers, this point is essentially at the free-surface and is also the maximum velocity in the vertical.

Inner Region

Very near the wall, viscosity dominates, so length scale must be a viscous length scale. We choose ν/u_* to form a viscous length scale

$$y_+ = \frac{y}{\nu/u_*} \tag{17}$$

Substituting this length scale into (13)

$$-\frac{\left(\overline{u'v'}\right)}{u_*^{\;2}} + \frac{d\left(\overline{u}/u_*\right)}{d\left(\dfrac{y}{\nu/u_*}\right)} = 1 - \frac{y}{\nu/u_*}\frac{\nu}{u_* h} \tag{18}$$

$$-\frac{\left(\overline{u'v'}\right)}{u_*^{\;2}} + \frac{d\left(\overline{u}/u_*\right)}{dy_+} = 1 - \frac{y_+}{Re} \tag{19}$$

For large *Re*, (19) becomes

$$-\frac{\left(\overline{u'v'}\right)}{u_*^{\;2}}+\frac{d\left(\overline{u}/u_*\right)}{dy_+}=1 \qquad (20)$$

Owing to the independence of the Reynolds stress and shear velocity from all parameters other than y (and thus y_+), each of the terms in (20) can be written

$$\frac{\overline{u}}{u_*}=f\left(y_+\right) \qquad (21a)$$

$$\frac{\overline{u'v'}}{u_*^{\;2}}=g\left(y_+\right) \qquad (21b)$$

These expressions are known as the **Law of the Wall**.

Very near the wall, the Reynolds stresses also become negligible. Therefore (20) becomes

$$y_+=\frac{\overline{u}}{u_*} \qquad (22)$$

This last assumption is usually valid for $y_+ < 5$. This extremely near-wall region is often referred to as the **Viscous Sublayer**.

Inertial Sublayer

Asymptotic analysis is used to match the velocity profile between the two regions. In the intermediate region, both the Law of the Wall and the Velocity-Defect Law must be applicable. We choose to match the derivative of the velocity profile (matching the value of the velocity tells you nothing).

$$\frac{d\bar{u}}{dy} = \frac{u_*}{h}\frac{dF}{d\eta} = \frac{u_*^2}{\nu}\frac{df}{dy_+} \qquad (23)$$

Rearranging and multiplying by y/u_*

$$-\frac{u_*}{h}\frac{dF}{d\eta} = \frac{u_*^2}{\nu}\frac{df}{dy_+} = \frac{y}{u_*}\frac{d\bar{u}}{dy} \qquad (24)$$

Asymptotic analysis tells us that if $\eta \to 0$ and $y_+ \to \infty$ simultaneously, the last term must be a constant. It turns out that experimental evidence suggested this would be true based upon an entirely different analysis of the 'mixing length' of the turbulence. Regardless of the original theory, we define κ

$$\frac{1}{\kappa} = \frac{y}{u_*}\frac{d\bar{u}}{dy} \qquad (25)$$

which is known as the von Karman constant.

With this, we are able calculate the matched velocity profile by integrating (24)

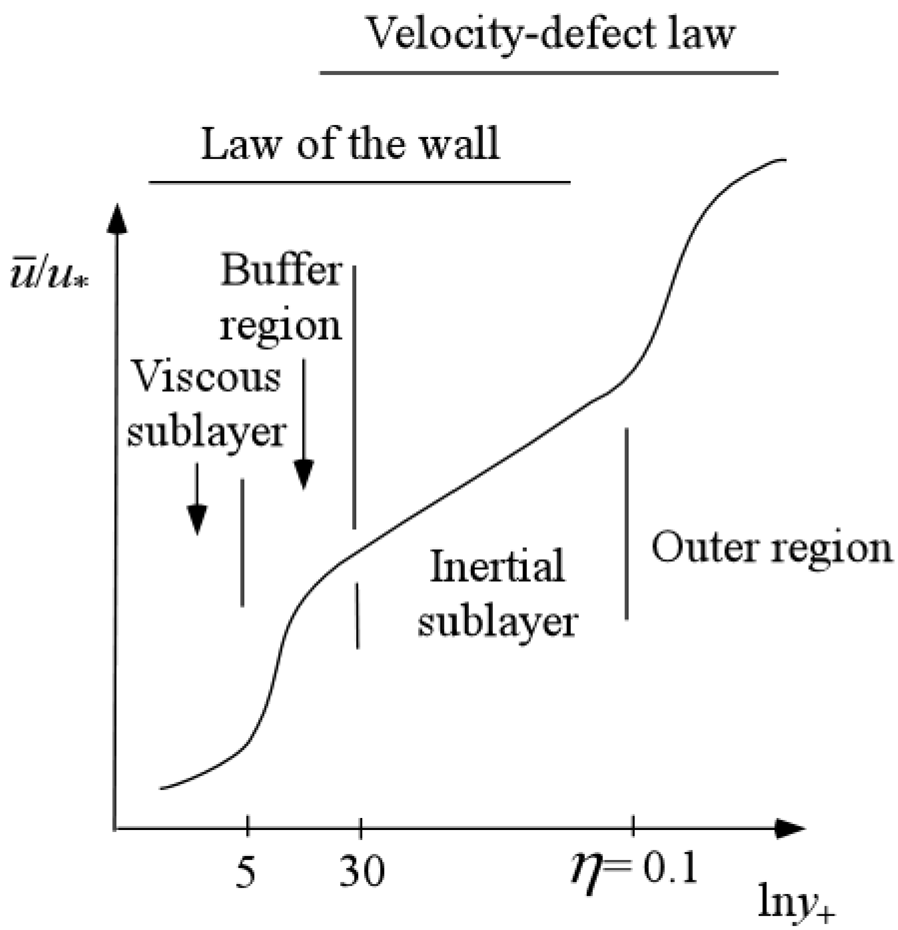
$$F(\eta) = \frac{\bar{u} - \bar{u}\big|_{y=h}}{u_*} = \frac{1}{\kappa}\ln\eta + a \qquad (25a)$$

$$f(y_+) = \frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln y_+ + b \qquad (25b)$$

The coefficients in (25a,b) must be determined experimentally. Innumerable studies have demonstrated $a \sim -1$ and $b \sim 5$.

Inertial sublayer (or log layer) typically extends from $y_+ > 30$ to $\eta < 0.1$. The intermediate region between the inertial sublayer and the viscous sublayer is often called the **Buffer Layer**. The location of maximum turbulence production is usually in the buffer layer.

In summary –

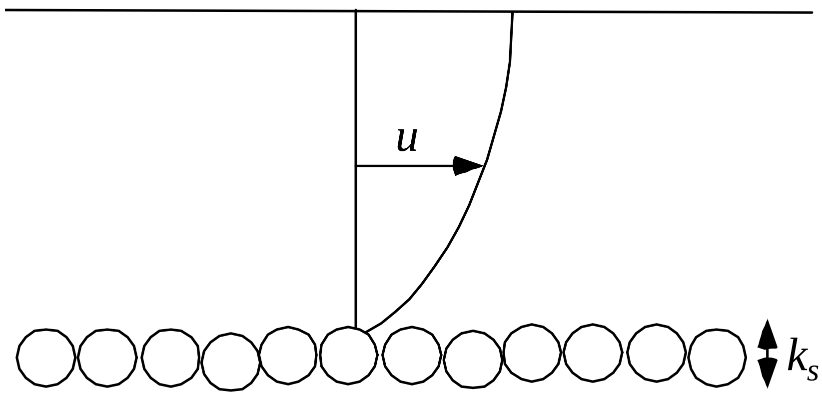


It should be noted that many assumptions were made along the way to the definition of the log-layer. They are:

- Little variation in the spanwise direction ($d/dz \sim 0$, formerly this was defined as y)
- Mean vertical and spanwise velocities are negligible ($v,w \sim 0$)
- Gradients in vertical are dominant ($d/dy >> d/dx$)
- Steady flow ($d/dt \sim 0$)
- Convective acceleration is negligible ($udu/dx \sim 0$)
- Relatively constant forcing ($dp/dx \sim C$)
- Far away from wall, but much smaller than the layer thickness ($y_+ > 30$, $\eta < 0.1$)

Rough channels

Most geophysical flows are rough. That is, the roughness on the bed is larger than the viscous sublayer.



If $k_s > 5\nu/u_*$, then the roughness will be larger than the viscous sublayer. At this point, viscous sublayer dynamics no longer regulates production of turbulence and the flow is called 'transitionally rough' ($k_s < 5\nu/u_*$, it is called "hydraulically smooth'). If $k_s > 30\nu/u_*$, the roughness elements extend into the inertial (log) sublayer. At this point, the flow is called 'hydraulically rough'.

For rough flows, the length scale ν/u_* is no longer applicable. We replace the roughness length scale k_s in the matching analysis above. This results in

$$f(y_+) = \frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\frac{y}{k_s} + c \qquad (26)$$

In this case, the coefficient in (26) again has to be determined. From numerous experiments, c = 8.5. There is inherently more variability in c than in b. This is mostly related to variations in the type of roughness.