

A Simple Formula to Estimate Settling Velocity of Natural Sediments

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Abstract: A simple formula to calculate the settling velocity of natural sediment particles for grain sizes between 0.063 and 1 mm is presented. The formula has been derived from the previous work of Dietrich, and it predicts the dimensionless settling velocity W_* as a function of a dimensionless fluid-sediment parameter S_* , provided the sediment shape factor and roundness are known. In case no information on shape and roundness factors is available, this paper recommends using the formula with a shape factor of 0.7 and a roundness value of 3.5 for naturally worn particles. The formula is tested against several independent data sets, and its performance is compared to other existing simple settling velocity formulas. For fine sediments with nominal diameters d_N between 0.063 and 0.25 mm, for which sediment suspension in natural conditions is most likely to occur, the proposed formula is shown to perform the best in terms of standard error of the estimation.

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Introduction

The settling velocity of sediment is one of the key variables in the study of sediment transport, especially when suspension is the dominant process, since it serves to characterize the restoring forces opposing turbulent entraining forces acting on the particle. For example, an error in the estimate of the settling velocity may be magnified by a factor of three or more in the computation of the suspended load transport. In spite of this importance, it is nearly impossible to obtain its actual value in situ, and in most cases it is obtained from laboratory experiments or predicted by empirical formulas.

Often the estimation of settling velocity of sediment has been done by applying predictive formulas developed by assuming the grains to be spheres (Gibbs et al. 1971). However, it is well known that the shape of natural sediment particles departs from a sphere. This departure will have some consequences, one being that the settling velocity will be lower than that of a sphere with the nominal diameter. Due to the practical implications of this difference, several formulas have been proposed to calculate the settling velocity of natural sediments [for example, Graf (1971); Hallermeier (1981); Dietrich (1982); van Rijn (1984); Cheng (1997); Ahrens (2000); and further references therein]. All of these have been empirically derived, and in this sense they fit very well the data set employed. However, as new relationships

have been proposed they have not incorporated previously published data to check the general accuracy of the respective formulas, and in this sense there is considerable uncertainty about which formula is the most accurate.

This paper presents a simple formula to estimate the settling velocity of natural sediment particles. The formula has been derived from the previous work of Dietrich (1982), and it predicts the settling velocity of a particle for a given sediment diameter, shape factor, and roundness. The objective is to obtain a formula based on a balance between simplicity and accuracy, that is, easy to use yet yielding accurate predictions. To test the performance of the proposed formula, several independent data sets have been compiled and used for this purpose as well as for comparison against other existing simple settling velocity formulas.

Development of Settling Velocity Formula

To estimate the settling velocity of sediment particles, two different approaches can be followed: (1) an idealized one in which the particle is assumed to be a sphere; and (2) a more realistic one in which the natural sediment shape is considered. In general, the first approach is used extensively (for instance, when the sediment grain size is obtained by using the settling tube, the grain size is calculated by assuming the sediment to be spherical), although some methods take into account the sediment shape.

Settling Velocity of Spherical Particle

The settling velocity of a sphere in a fluid at rest can be estimated by solving the balance between the gravitational force and the drag resistance:

$$(s-1)\rho g \frac{\pi}{6} d^3 = \frac{1}{2} \rho C_D \frac{\pi}{4} d^2 w_s^2 \quad (1)$$

where s = specific gravity of the sediment given by ρ_s/ρ ; ρ_s and ρ = densities of the sediment and the fluid, respectively; g = gravitational acceleration; d = sediment diameter; C_D = drag coefficient; and w_s = settling velocity.

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Rearranging Eq. (1), this may be written in terms of a dimensionless settling velocity:

$$W_* = \frac{w_s}{\sqrt{(s-1)gd}} = \sqrt{\frac{4}{3C_D}} \quad (2)$$

in which C_D is a function of the Reynolds number:

$$R = \frac{w_s d}{\nu} = 4W_* S_* \quad (3)$$

with ν being the kinematic viscosity of the fluid and S_* , the fluid-sediment parameter introduced by Madsen and Grant (1976), given by

$$S_* = \frac{d}{4\nu} \sqrt{(s-1)gd} \quad (4)$$

For low values of the Reynolds number ($R < 1$), Stokes law is valid and $C_D = 24/R$, whereas $C_D \cong \text{constant} \cong 0.4$ for $10^3 < R < R_{\text{crit}} \cong 3 \cdot 10^5$ [for example, Schlichting (1960)]. Introducing these asymptotic expressions in Eqs. (2) and (3) leads to the limiting values for the dimensionless settling velocity of spherical particles

$$W_* = \begin{cases} S_*/4.5 & S_* < 1 \\ 1.83 & 150 < S_* < 4 \cdot 10^4 \end{cases} \quad (5)$$

In the context of quartz spheres in water, the limiting expressions given by Eq. (5) correspond to diameters $d < \sim 0.1$ mm ($S_* < 1$) and ~ 3 mm $< d < \sim 12$ cm. Thus, the important range corresponding to sand-sized sediments in water falls in the transitional range $1 < S_* < 150$, for which no straightforward analytical solution exists. As an example, Dietrich (1982) derived an expression to calculate the settling velocity of spheres for the full range of R (or equivalent S_*) by fitting a fourth-order polynomial to a data set consisting of 252 values.

Guided by the asymptotic behavior expressed by Eq. (5), we adopt in the present study a general expression for the settling velocity of a sediment grain

$$\frac{1}{W_*} = A + \frac{B}{S_*} \quad (6)$$

in which A and B are constants to be obtained by fitting Eq. (6) to experimental data for a certain range of S_* values.

The adopted form for the empirical relationship for the dimensionless settling velocity, Eq. (6), is a compromise between accuracy and simplicity. In addition to simplicity, the simple expression given by Eq. (6) has the advantage over a more accurate three-term expression, proposed in an earlier version of this paper, in that Eq. (6) allows for a solution for the drag coefficient, C_D , as a function of the Reynolds number, R . This solution

$$C_D = \frac{1}{3} (A + \sqrt{A^2 + 16B/R})^2 \quad (7)$$

is obtained from Eq. (6) by expressing W_* in terms of C_D using Eq. (2), and S_* in terms of R using Eq. (3).

As an example, Eq. (6) was fitted, by linear regression of $1/W_*$ against $1/S_*$, to Dietrich's (1982) empirical expression for the nondimensional settling velocity of spheres for the range $0.5 < S_* < 30$, corresponding to quartz spheres of ~ 0.06 mm $< d < \sim 1$ mm in water, to obtain $A = 0.79$ and $B = 4.61$ for the constants in Eq. (6). Corresponding to the upper limit of $S_* = 30$, we obtain

$W_* = 1.06$ from Eq. (6), and consequently an upper bound for $R = 127$ from Eq. (3). Similarly, a lower bound for $R = 0.2$ is obtained for $S_* = 0.5$. Thus, the expression for C_D given by Eq. (7) is valid for a Reynolds number range $0.2 < R < 127$.

It is of interest to note that extension of Eq. (7) beyond its region of validity gives $C_D = 24.5/R$ for $R \rightarrow 0$, in excellent agreement with Stokes law, and $C_D = 0.83$ for $R \rightarrow \infty$, which is significantly different from the constant value of $C_D = 0.4$ obtained experimentally for $R > 10^3$. Thus, extrapolation of Eq. (6) beyond the lower range of S_* for which the constants A and B were determined may be considered allowable, whereas extrapolation beyond the upper limit, in general, may lead to significant errors. This behavior of Eq. (6) is, of course, a consequence of obtaining the values of A and B from a linear regression of $1/W_*$ against $1/S_*$ which favors fitting large values of these variables, that is, small values of W_* and S_* , corresponding to smaller grain sizes. Since suspended sediment transport is more likely to be important for smaller sediment sizes, the relatively better fit for smaller grain sizes at the expense of a relatively poorer fit for larger grain sizes may be considered a desirable feature of our formula.

Setting Velocity of Natural Sediments

When natural sediments are considered, another variable must be included since the relationship between C_D and R will be affected by the particle shape [for example, Graf (1971)]. The deviation of a particle's shape from a sphere is generally quantified by a shape factor. The most commonly used is Corey's shape factor (csf), which is given by $\text{csf} = c/(ab)^{0.5}$ where a , b , and c are the longest, intermediate, and shortest axes of the particle. This shape factor takes a zero value for a 2D plate and unity for a sphere and represents the ratio of the cross-sectional area of an inscribed sphere to the maximum cross-sectional area of an ellipsoidal particle. Further details on different shape factors can be found in Alger and Simmons (1968).

Since the most stable orientation of irregular grains is to have their maximum projected area oriented perpendicular to the direction of fall, this implies that the fluid will be displaced around a larger surface in comparison with a sphere of the same nominal diameter [for example, Graf (1971); Dietrich (1982); and references therein]. This will induce flow separation around the grain and consequently an increase in the drag. The practical result is that the settling velocity of a natural (nonspherical) grain will be lower than that of a sphere with the nominal diameter, that is, with the same volume as the natural grain.

A typical way to account for the shape effect is to obtain the limiting values of C_D at low and high R (or equivalent S_*) and to look for a functional relationship for C_D in the intermediate region connecting both ends by fitting experimental data over this range. Different connection functions can be found in Swamee and Ojha (1991a,b) and Cheng (1997), among others. These C_D functions depend not only on R and on shape factor csf, but also on the particle roundness P , which is usually considered by analyzing naturally rounded and crushed particles [for example, Schulz et al. (1954); Dietrich (1982); Swamee and Ojha (1991a, b)]. This roundness is a measure of the curvature variations along the grain surface, and it is usually estimated from the measurement of angular deviations from a circle inscribed in the maximum projected area or on a plane perpendicular to the shortest axis [for example, Janke 1966].

Dietrich (1982) developed an expression accounting for the effects of shape and roundness on the settling velocity. In his expression, the sediment grain is defined by its nominal diameter

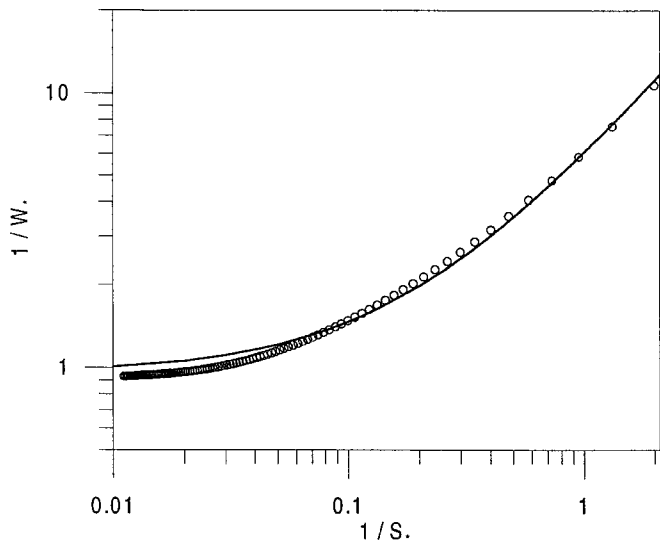


Fig. 1. Fitting of functional relationship in Eq. (6) (full line) to predictions of Dietrich's formula (open circles) for natural grains with standard shape and roundness factors ($csf=0.7$ and $P=3.5$)

d_N (the diameter of a sphere of the same volume as the natural particle), its shape factor csf , and roundness P . The formula was obtained from a data set of 245 smooth well-rounded particles ($P=6$), 538 values for naturally rounded particles ($P=3.5$), and 111 values for crushed sediments ($P=2.0$) and can be considered valid for shape factors between $[0.4,0.9]$ and for S_* values up to 17,700. Although Dietrich's (1982) analysis should be regarded as the most comprehensive to date, in terms of the number of data employed to derive the formula, it is, with a few exceptions [for example, McLean (1992)], seldom employed in practice. The reasons for this may be associated with its relatively complex form due to the large number of terms included in Dietrich's formula.

In this paper we adopt the simple expression given by Eq. (6) and obtain the values of A and B by fitting Dietrich's formula for the range $0.5 < S_* < 30$, with all dimensionless variables defined by Eqs. (2) through (4) evaluated using the nominal diameter d_N for d . The adopted range of S_* corresponds to quartz sands of nominal diameters in the range $\sim 0.06 \text{ mm} < d_N < \sim 1 \text{ mm}$ settling in water. Below this range, sediment sizes are generally obtained from settling tube analysis, that is, the reported diameter is the fall diameter, obtained from Stokes law. Above this range, significant sediment transport in suspension is highly unlikely. Thus, the chosen range should cover most cases encountered when sediment transport in suspension may be important.

Fig. 1 shows the fit obtained by Eq. (6) to Dietrich's formula for standard values of roundness and shape factors, $P=3.5$ and $csf=0.7$, respectively, corresponding to naturally worn sediments. Although the fit was obtained for the range $S_* < 30$ ($1/S_* > 0.03$), the comparison shown in Fig. 1 is extended to $S_* = 100$ ($1/S_* = 1/100$) to illustrate the potential error associated with such an extrapolation beyond the limit of validity of Eq. (6). Fig. 2 presents the values of the coefficients A and B in Eq. (6) for the range of roundness $2 < P < 6$, with $P=3.5$ being the standard value, and shape factors $0.4 < csf < 0.9$, with $csf=0.7$ being the standard value covered by the data used by Dietrich to establish his formula. Since increased angularity of a particle, that is, decreasing roundness, is expected to lead to an increase in drag coefficient, it is encouraging to note that A and B generally decrease with increasing P and therefore result in an increase in

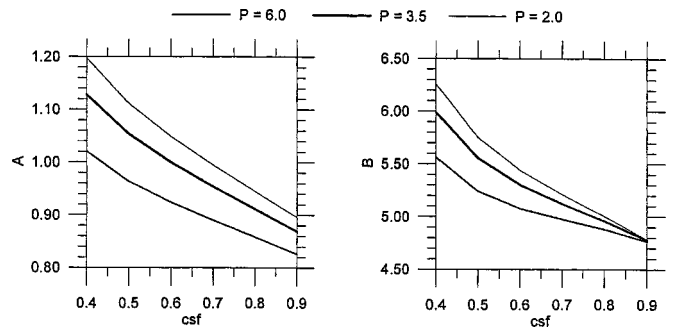


Fig. 2. Variation of coefficients (A, B) in Eq. (6) with shape factor and roundness

settling velocity as the particles become rounder. Similarly, A and B are seen to decrease with increasing shape factor, which again confirms the expectation that the settling velocity of a particle increases the more it resembles a sphere.

In addition to providing the ability to obtain the coefficients A and B for a range of P and csf values for use in Eq. (6) to predict settling velocities, the same values may be used in Eq. (7) to obtain drag coefficients for sediment grains when P and csf are known.

To quantify the degree to which Eq. (6) fits Dietrich's formula, we defined the "error" as

$$\varepsilon = \left(\frac{w_{s,c}}{w_{s,m}} - 1 \right) 100 \quad (8)$$

where $w_{s,c}$ is the calculated settling velocity, here obtained from Eq. (6); and $w_{s,m}$ is the measured settling velocity, here the prediction afforded by Dietrich's formula. Table 1 presents the best fit values for A and B for $P=2, 3.5$ and 6 and $csf=0.7$ obtained by fitting Dietrich's formula and the associated statistics of the error defined by Eq. (8) in terms of its mean value, μ , equivalent to a bias, and its root-mean-square value, ε_{rms} .

Evaluation of Settling Velocity Formula's Performance

For nonspherical grains, several empirical formulas exist to estimate the settling velocity of natural sediments [for example, Graf (1971); Zanke (1977); Hallermeier (1981); Dietrich (1982); van Rijn (1984); Swamee and Ojha (1991b); Julien (1995); Cheng 1997; Soulsby (1997); Ahrens (2000); and references therein].

Table 1. Coefficients in Eq. (6) for Spherical Grains and Natural Sediments with Shape Factor of 0.7 and Roundness Values P , Typical of Crushed (2.0), Natural (3.5), and Well-Rounded (6.0) Sediments, Goodness of Fit (r^2 = Coefficient of Determination), and Error (μ and ε_{rms}) for $0.5 < S_* < 30$

P -value	A	B	r^2	μ	ε_{rms}
2.0	0.995	5.211	0.9972	-1.0	4.1
3.5	0.954	5.121	0.9968	-1.1	4.8
6.0	0.890	4.974	0.9957	-1.4	5.9
Spheres	0.794	4.606	0.9954	-1.6	6.5

Table 2. Comparison, in Terms of Error Defined by Eq. (8), between Predicted Settling Velocities According to Different Methods against Data Generated by Applying Dietrich (1982) for $csf=0.7$ and $P=3.5$ (Data Have Been Generated in Range of Diameters [0.063–1.0] mm of Quartz in Water) with Interval of 10 μm , Equivalent to S_* Range [0.50–31.81]

Code		Jiménez and Madsen (2003)	Zanke (1977)	Julien (1996)	Soulsby (1997)	Cheng (1997)	Ahrens (2000)
All	μ	-1.14	0.40	0.15	2.24	-6.03	0.29
	ϵ_{rms}	4.79	8.44	11.11	8.72	6.97	5.17
Very fine and fine (0.063–0.25)	μ	2.91	4.13	11.76	5.43	-4.89	8.49
	ϵ_{rms}	4.25	9.61	13.79	10.45	6.53	9.71
Medium (0.25–0.50)	μ	3.35	9.60	9.73	11.56	-2.27	1.38
	ϵ_{rms}	3.80	10.15	10.91	12.00	2.66	2.77
Coarse (0.50–1.00)	μ	-4.86	-5.50	-8.87	-3.51	-8.30	-3.28
	ϵ_{rms}	5.37	6.92	10.39	5.50	8.44	3.31

Note: (d_i-d_f) Diameter range in millimeters.

Zanke (1977); Julien (1995); and Soulsby (1997) developed three similar formulas to predict the settling velocity of natural grains, which when expressed in the present notation may be written

$$W_* = \frac{\alpha}{S_*} [(1 + \beta S_*^2)^{0.5} - 1] \quad (9)$$

The differences among these formulas are given by the coefficients (α, β), which take the values (2.5, 0.16) for Zanke (1977), (2.0, 0.222) for Julien (1995), and (2.59, 0.156) for Soulsby (1997). The different values of the coefficients are due to the different data sets used in their respective empirical derivations. The sediment diameter in these formulas is the sieve diameter d_s .

Recently, Cheng (1997) proposed a simple formula that, in the present notation, reads

$$W_* = \frac{1}{4S_*} (\sqrt{25 + 7.6S_*^{4/3}} - 5)^{1.5} \quad (10)$$

to calculate the settling velocity of natural sediments by estimating the relation between C_D and R from a data set of 43 values in the intermediate Reynolds number range, $1 < R < 1,000$. One of the differences between Cheng's approach and Dietrich's is that Cheng's does not explicitly account for shape factor or roundness value. It assumes that all natural sediments have a "standard" shape factor of 0.7, and, moreover, the sediment diameter is taken to be the arithmetic average rather than the nominal diameter (Cheng 1998).

Finally, we have included a recent formula in this analysis:

$$W_* = C_1 + C_2 S_* \quad (11a)$$

$$C_1 = 1.06 \tanh(0.064 S_* \exp(-7.5/S_*^2)) \quad (11b)$$

$$C_2 = 0.22 \tanh(2.34 S_*^{-1.18} \exp(-0.0064 S_*^2)) \quad (11c)$$

proposed by Ahrens (2000) and expressed here in terms of our variables. It was developed by fitting Hallermeier's (1981) expressions by a single mathematical formula instead of using different expressions for two ranges of sediment sizes as done in the original method. The formula does not explicitly account for shape factor or roundness value. The sediment diameter used by Ahrens was the sieve diameter d_s .

Although Ahrens's formula as given by Eq. (11a) appears to be as simple as the other formulas considered here, his more elaborate expressions for the coefficients C_1 and C_2 , given by Eqs. (11b and 11c), makes his formula somewhat more complex than

Eqs. (6), (9), and (10). Thus, Ahrens's formula may be considered a level above the others in terms of sophistication (and a level below in terms of simplicity). This feature of his formula should be kept in mind when the skills of the various settling velocity formulas are compared in subsequent sections of this paper.

Settling Velocity Formulas Comparison

Assuming that Dietrich's formula is an exact representation of the data set he employed in its derivation, all the above-mentioned formulas were tested against its prediction. In order to make this comparison meaningful, the difference in "diameters" employed in different formulas must be resolved. To achieve this, we utilized the rule of thumb $d_s/d_N \cong 0.9$, suggested by Raudkivi (1990), to convert sieve to nominal diameter and assumed the arithmetic mean diameter, used by Cheng (1997), to equal the nominal diameter.

The performance of the different formulas was measured by the statistics of the error defined by Eq. (8) for ranges of diameters of quartz sand ($P=3.5$ and $csf=0.7$) in water corresponding to very fine and fine sands (0.063–0.25 mm), medium sands (0.25–0.50 mm), and coarse sands (0.50–1.00 mm). The present formula, Eq. (6), was based on the best fit values given in Table 1.

Since the present formula, Eq. (6), was obtained by fitting to Dietrich's formula—that is, the data against which all formulas are tested—it is not surprising that Eq. (6) is seen from Table 2 to perform better than the rest for the full range of S_* considered. It is clearly outperforming the family of formulas given by Eq. (9), but is only marginally better than the formulas of Cheng (1997), Eq. (10), and Ahrens (2000), Eq. (11), over the full range of diameters. However, the present formula is clearly superior to the rest for the range corresponding to very fine and fine sands, that is, the range for which sediment suspension and hence an accurate prediction of the settling velocity is of greatest importance.

In addition, and to put in context the errors calculated according to Eq. (8), the ratio of calculated to "measured," that is, the prediction afforded by Dietrich (1982), settling velocity was computed for each formula and plotted in Fig. 3 to illustrate graphically the nature of the inaccuracy associated with each formula. In the very fine and fine sands range, the formula proposed here predicts values within 5% of those measured. The Zanke (1997); Julien (1995); and Soulsby (1997) formulas predict most of the corresponding values with deviations much larger up to 15%, with the maximum deviation located at $d_N=0.25$ mm. It has to be stressed that the similarity of the mean errors, that is, relative

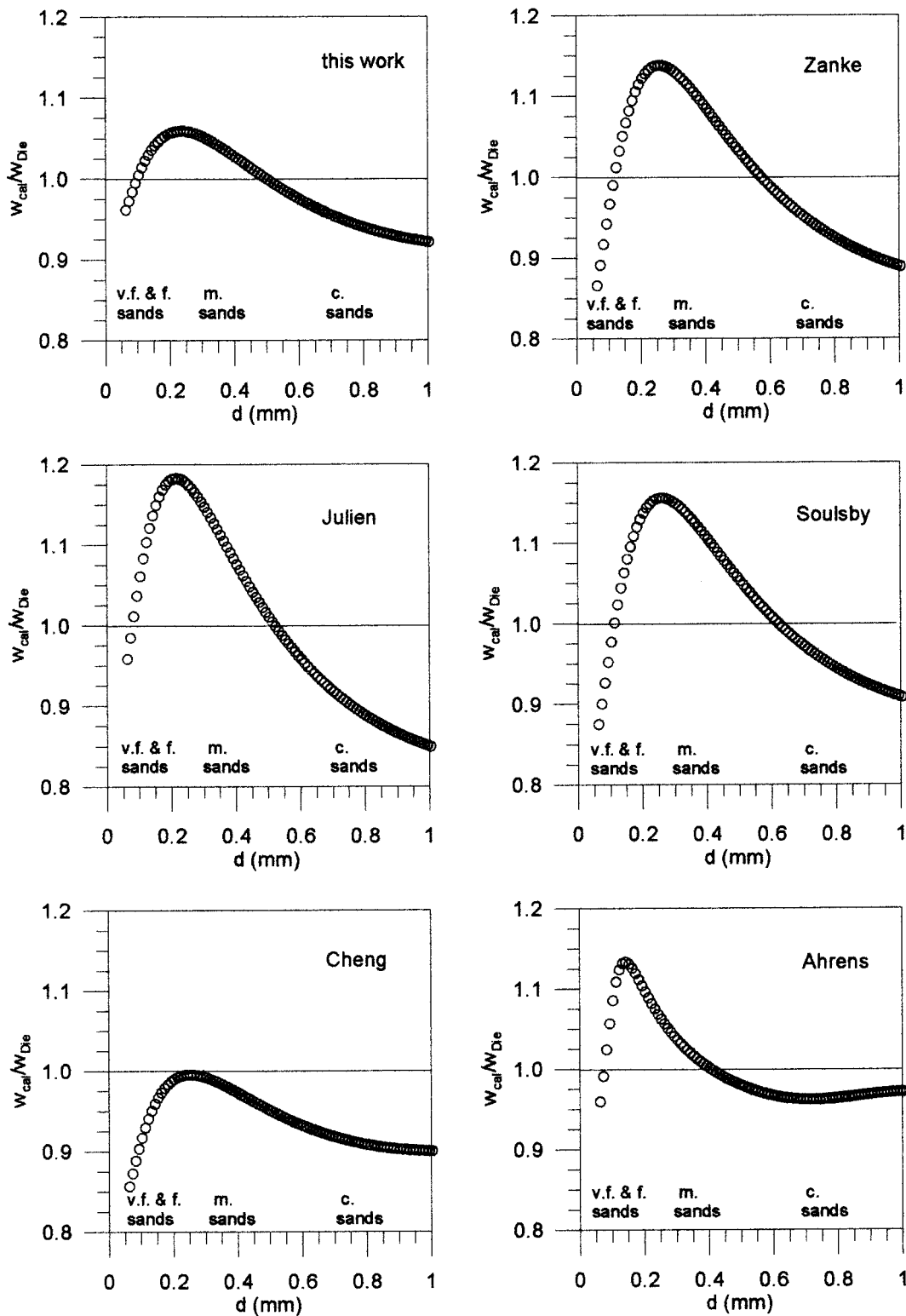


Fig. 3. Ratio of prediction of settling velocity according to each formula, w_{cal} , to prediction afforded by Dietrich's (1982) formula, w_{Die} (*v.f. & f.* = very fine and fine; *m.* = medium; *c.* = coarse sands)

bias, presented in Table 2 of Zanke's and Soulsby's formulas to the one proposed here is because the predictions according to these formulas, although showing larger deviations (up to 15%), are "symmetrical" with respect to the measured value, that is, large overpredictions are counterbalanced by large underpredictions. Cheng's (1997) formula generally underpredicts the measured fall

velocity with maximum deviations up to 13% for the very fine sand fraction ($d_N < 0.125$ mm) and within 5% for the fine sand fraction. Ahrens's (2000) formula overpredicts the measured values up to a maximum of 13%, which is attained at $d_N = 0.125$ mm. In the medium sand range, Ahrens's and Cheng's formulas and the formula proposed here predict fall velocities

Table 3. Data Used for Settling Velocity Formulas Validation

Code	Data	<i>n</i>	S_*		d (mm)		w_s (m/s)	
			Range	Mean	Range	Mean	Range	Mean
1	Cheng (1997) ^{a,e}	37	0.42–219.7	45.23	0.061–4.5	1.15	0.002–0.281	0.101
2	Engelund and Hansen (1972) ^{d,e}	22	0.77–83.30	16.22	0.100–1.9	0.580	0.005–0.170	0.063
3	Engelund and Hansen (1972) ^{e,g}	22	0.64–76.81	14.54	0.089–1.8	0.540	0.005–0.170	0.063
4	Hallermeier (1981) ^{e,g}	20	1.62–12.97	6.48	0.137–0.55	0.332	0.017–0.075	0.045
5	Hallermeier (1981) ^{b,d,e}	20	1.89–15.19	7.59	0.152–0.61	0.369	0.017–0.075	0.045
6	Raudkivi (1990) ^{c,d}	12	2.17–99.09	32.43	0.200–2.0	0.930	0.017–0.156	0.082
7	Raudkivi (1990) ^{c,f}	12	2.17–99.09	32.43	0.200–2.0	0.930	0.018–0.194	0.098
8	Raudkivi (1990) ^{d,h}	12	2.17–99.09	32.43	0.200–2.0	0.930	0.019–0.240	0.116

^aArithmetic average diameter (d_n).

^bAssuming $d_s/d_N \sim 0.9$.

^cShape factor=0.5.

^dNominal diameter (d_N).

^eShape factor not given.

^fShape factor=0.7.

^gSieving diameter (d_s).

^hShape factor=0.9.

within 5% of the measured values, whereas the predictions of the other formulas are above this limit. Finally, in the coarse sand range, Ahrens's formula performs the best, followed by the formula proposed here.

Validation of Settling Velocity Formula

To validate the proposed formula, Eq. (6), and to compare its performance against the other formulas considered, different published data sets have been used in a validation test. This validation is done by comparing the predicted settling velocity against measured values in terms of the error of the estimation, as introduced and defined by Eq. (8).

Two tests have been performed: (1) one that employs data sets in which the shape factor was not given and sediment is characterized as being composed by natural grains; and (2) one in which the shape factor is also provided.

Data Sets

The first group of data sets corresponds to settling velocities of natural sediments without an explicit definition of the shape factor and were taken from Cheng (1997); Engelund and Hansen (1972); and Hallermeier (1981) (Table 3) where the number of data points

from each source, n , is listed in the third column. In all cases it has been assumed (by the respective writers, and here we follow their approach) that the corresponding shape factor can be represented by a value of $csf=0.7$, which is usually taken as the most common value for naturally shaped sediments [for example, Dietrich (1982)].

The Cheng (1997) data set is a compilation of Russian quartz sand experiments (original references can be found in Cheng's paper) in which the sediment size was characterized through the arithmetic average diameter (Cheng 1998). Because the specific gravity s was not given, it was assumed to be 2.65. The kinematic viscosity of the fluid was calculated as corresponding to fresh water at the specified temperature.

The Engelund and Hansen (1972) data set was taken from Fredsøe and Deigaard (1992). The sediment size was characterized through the sieve diameter d_s , and the nominal diameter d_N and settling velocities were measured at 10°C and 20°C. Because the specific gravity s was not given, it was assumed to be 2.65. The kinematic viscosity of the fluid was calculated corresponding to fresh water at the specified temperature.

Since the formula proposed here was derived to be used with the nominal diameter, the given sieve diameters were, as previously mentioned, converted to nominal diameter by using the rule of thumb $d_s/d_N \cong 0.9$ (Raudkivi 1990). As an example of the applicability of this approach, the ratio calculated from the data

Table 4. Comparison, in Terms of Error Defined by Eq. (8), or Predicted and Measured Settling Velocities According to Different Methods for Natural Sediments (No Shape Factor Given and Assumed to be 0.7; Data Characteristics Are Given in Table 3).

Code		Jiménez and Madsen (2003)	Zanke (1977)	Julien (1996)	Soulsby (1997)	Cheng (1997)	Ahrens (2000)
all	μ	-1.17	0.42	1.34	2.17	-5.82	1.05
	ϵ_{rms}	10.80	12.29	14.62	12.51	11.44	10.85
all (0.063–0.25)	μ	1.09	2.15	9.34	3.45	-6.60	5.44
	ϵ_{rms}	14.02	14.08	17.18	14.55	14.23	15.19
all (0.25–0.50)	μ	0.72	8.60	8.37	10.57	-4.68	1.04
	ϵ_{rms}	9.40	12.20	12.62	13.73	10.11	7.80
all (0.50–1.00)	μ	-4.90	-3.70	-6.78	-1.71	-8.81	-3.75
	ϵ_{rms}	10.63	9.35	10.75	8.93	12.22	9.59
all (>1.00)	μ	-4.25	-10.57	-15.26	-8.56	-3.10	-2.06
	ϵ_{rms}	5.38	11.19	15.65	9.34	5.09	4.79

Note [$di-df$]=diameter range in millimeters.

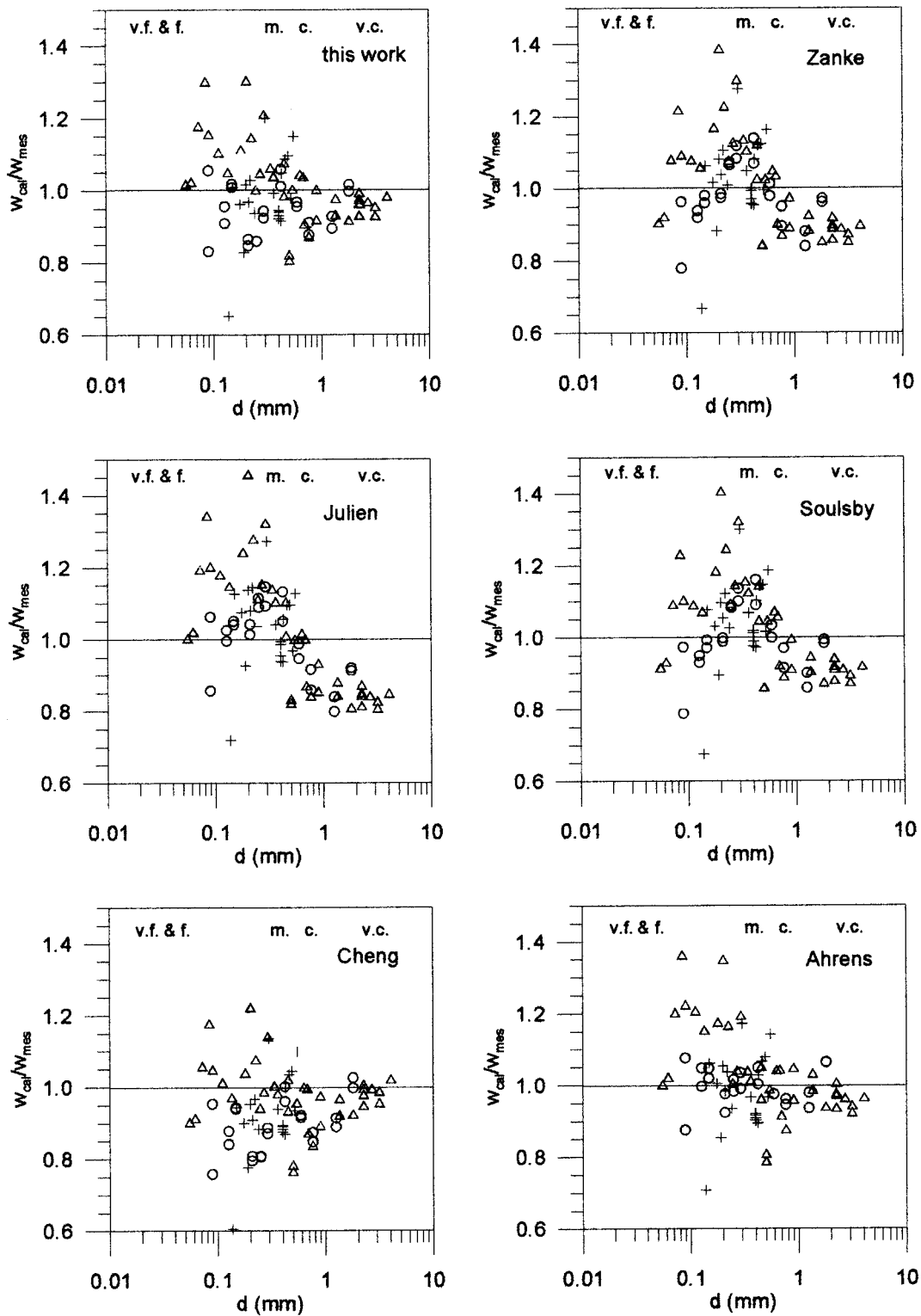


Fig. 4. Ratio of predicted and measured settling velocities according to each formula considered. Data sources, also indicated in Table 3, are identified by the symbols: \circ , Engelund-Hansen; $+$, Hallermeier; Δ , Cheng; with size classes specified by *v.f. & f.*, very fine and fine; *m.*, medium; *c.*, coarse; *v.c.*, very coarse sands.

supplied by Engelund and Hansen (1972) gives a mean value of 0.93 with a standard deviation of 0.04.

The Hallermeier (1981) data set is a compilation of previously published experiments (original references can be found in Hallermeier's paper), in which the sediment size was characterized by the sieve diameter. We have restricted the analysis to sands in the quartz range, so only experiments with a specific

gravity between 2.57 and 2.67 were considered. The kinematic viscosity was given for some experiments, and when it was not, a value of $10^{-6} \text{ m}^2/\text{s}$ was assumed (Hallermeier 1981). The original compilation of Hallermeier's data set also included the Engelund and Hansen (1972) data, but we are considering it separately.

The second group of data sets corresponds to settling velocities reported by Raudkivi (1990, p. 20), originally given by the

Table 5. Prediction Score in Settling Velocity Estimation for Overall Data Set Measured as Percentage of Cases within Specified Error of Prediction (Calculated/Measured) for Each Formula

Percentage	Jiménez and Madsen (2003)		Julien (1996)	Soulsby (1997)	Cheng (1997)	Ahrens (2000)
	Zanke (1977)					
10	72	56	46	62	62	77
20	94	91	89	91	91	91

U.S. Inter-Agency Committee. The reported data consisted of settling velocities of sediment characterized by its nominal diameter and shape factor (Table 3). Specific gravity and water temperature were also provided.

Results

Table 4 shows the error, as defined by Eq. (8), of the prediction afforded by each formula for the measured data listed in Table 3, for which no shape factors were reported. Fig. 4 shows the ratios of predicted and measured velocities.

Prior to a discussion of the comparison of the performance of the different formulas exhibited in Table 4 and Fig. 4, it is stressed that about 50% of the data used here were used in the derivation of the formulas by Soulsby (1997); Cheng (1997); and Ahrens (2000), whereas none of the data were used in the derivation of the present formula, Eq. (6). Thus, in contrast to the comparison against Dietrich's formula, Table 2 and Fig. 3, the present comparison should favor the other formulas rather than Eq. (6).

Considering the entire range of sediment diameters included in the data set used, all formulas give similar accuracy, with Cheng's (1997) formula exhibiting the largest bias, but an RMS error similar to the rest. Another measure of overall performance of the different formulas is the percentage of cases predicted within a specified accuracy range around the measured value. This percentage (score) is listed in Table 5 and shows the similar and somewhat superior performance of the present and Ahrens's (2000) formulas.

Separating the comparison into size classes of the sediment, Table 4 shows that the present formula outperforms the rest for very fine and fine sands only in terms of its smaller relative bias, whereas its RMS error is insignificantly different from most of the other formulas. This is a somewhat surprising result in light of the present formula's superior performance for very fine and fine sands when compared to Dietrich's (1982) results (Table 2). However, equally surprising is the present formula's performance for coarse and very coarse sands. It is particularly gratifying to notice that application of Eq. (6) for sediment diameters larger than 1 mm, which is outside the expected region of validity of Eq. (6) since this was established by fitting Dietrich's formula for $S_* < 30$ ($d < 1$ mm), provides settling velocity predictions rivaling those of the best performers (Ahrens and Cheng) in this range. Again, to place this comparison in context, it is emphasized that the only formula not including any of the test data used is the one derived in this paper.

Finally, the proposed formula, Eq. (6), was tested against the Raudkivi data set in which the shape factor of the sediment was specified (Table 3), with A and B obtained corresponding to the standard roundness factor $P=3.5$, from Fig. 2. The other formulas were not used because they do not allow the user to specify a shape factor. As can be seen from Table 6, the proposed method

Table 6. Comparison, in Terms of Error Defined by Eq. (8), of Predicted and Measured Settling Velocities According to Proposed Formula When Sediment Shape Factor is Specified [data from Roudkivi (1990)]

csf	Error		
	n	μ	ϵ_{rms}
0.5	12	1.63	3.49
0.7	12	-3.72	5.05
0.9	12	-7.70	10.39
all	36	-3.26	6.97
0.5(*)	12	11.67	12.27
0.9(*)	12	-15.27	16.71

Note: (*) calculated assuming csf=0.7.

predicts the settling velocity for different shape factors reasonably well, with mean errors of 2–4% for a csf of 0.5 and 0.7–8% in the case of a csf of 0.9. The mean error for the overall data set is 3%.

To assess the potential deviation in the estimated settling velocity of natural sediments with shape factors different from 0.7 by assuming this value to be representative (as most of the formulas do), Table 6 also shows the comparison of measured settling velocities for csf values of 0.5 and 0.9 against those predicted by using csf=0.7. As can be seen, the use of the "common" value (0.7) significantly increases the deviations of the predicted value with respect to those measured, with a mean increase in the error to 12 and 15% for csf values of 0.5 and 0.9, respectively. This serves to stress the necessity to have a formula capable to deal with different shape factors to properly estimate the settling velocity of natural sediments with variable shape factors. For this purpose, A and B coefficients to be used in Eq. (6) for any shape factor can be obtained from Fig. 2.

Summary

A simple formula for the computation of the settling velocity w_s of particles of density $\rho_s = s\rho$ and nominal diameter d_N , in a quiescent fluid of density ρ and kinematic viscosity ν , has been developed by fitting the formula of Dietrich (1982) to the expression

$$W_* = \frac{w_s}{\sqrt{(s-1)gd_N}} = \left(A + \frac{B}{S_*} \right)^{-1} \quad (12)$$

in which

$$S_* = \frac{d_N}{4\nu} \sqrt{(s-1)gd_N} \quad (13)$$

While far simpler and more user friendly than the original formula proposed by Dietrich, the proposed formula retains the generality of the original by providing values of the coefficients A and B , in Eq. (12) as functions of particle shape factor csf and Powers's roundness P (Fig. 2); that is, it affords the ability to predict settling velocities that accounts for the influence of particle shape and roundness, if these are known. In many practical applications the sediment is likely to be naturally worn quartz sands characterized only by their sieve diameter d_s . For this typical application, $d_N = d_s/0.9$ and standard values of $A = 0.954$ and $B = 5.12$, corresponding to csf=0.7 and $P = 3.5$, are proposed.

Although values of A and B (Fig. 2) are obtained by fitting Dietrich's formula for a range of $0.5 < S_* < 30$, corresponding to a diameter range of $\sim 0.063 \text{ mm} < d_N < \sim 1.0 \text{ mm}$ for quartz in water, the proposed formula's typical application is found to yield better or comparable predictions than other simple settling velocity formulas for the entire sand range, that is, up to $d_N \cong 2 \text{ mm}$. This is demonstrated through comparison with measured fall velocities that were not used by Dietrich in the derivation of his formula. From this comparison the typical application of Eq. (12) shows a negligible relative bias of the predictions (about 1%) and an RMS error; equivalent to a coefficient of variation, of about 10% (Table 4). Thus one can expect predicted settling velocities from Eq. (12) for naturally worn sands to fall within 10% of the correct value in roughly 70% of the applications.

The utility of the simple formula's [Eq. (12)] ability to account for shape factors when these are different from the standard value of $\text{csf} = 0.7$ is demonstrated (Table 6) by the vastly improved predictions when A and B are obtained for the actual shape factors ($\text{csf} = 0.5$ and 0.9) rather than the predictions afforded by Eq. (12) with the standard values of A and B , corresponding to $\text{csf} = 0.7$.

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Notation

The following symbols are used in this paper:

- A = dimensionless zero-order coefficient in Eq. (6);
- a = largest length of sediment particle;
- B = dimensionless first-order coefficient in (6);
- b = intermediate length of a sediment particle;
- C_D = drag coefficient;
- c = shortest length of a sediment particle;
- csf = Corey's shape factor $c/(ab)^{0.5}$;
- d = sediment diameter;
- d_N = nominal diameter;
- d_s = sieving diameter;
- g = gravitational acceleration;
- P = particle roundness;
- R = Reynolds number;
- r^2 = coefficient of determination;
- S_* = dimensionless particle parameter;
- s = specific gravity ($s = \rho_s / \rho$);
- W_* = dimensionless settling velocity;
- w_s = settling velocity;
- α = coefficient in Eq. (9);
- β = coefficient in Eq. (9);

- ε = error in the estimation = $[(w_{s \text{ cal}} / w_{s \text{ mes}}) - 1]100$;
- ε_{rms} = root-mean-square value of ε ;
- μ = mean value of ε ;
- ν = kinematic viscosity of fluid;
- ρ = density of fluid; and
- ρ_s = density of sediment.

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