Generalized Particle Motion in a Moving Fluid

The equations that describe the momentum of a solid particle moving through a dynamic fluid are quite complex. Wiberg and Smith (1985) describe

$$\rho_s V_p \frac{d\vec{u}_p}{dt} = -\rho_s V_p \vec{g} + \int_S \mathbf{T} \cdot \vec{n} dS \qquad (6a)$$

where V_p is the volume of the particle, ρ_s is the density of the particle, u_p is the particle velocity vector, and $\mathbf{T}$ is the instantaneous stress tensor that must satisfy the N-S equations

$$\frac{D\vec{u}}{Dt} = \vec{g} + \frac{\nabla \cdot \mathbf{T}}{\rho_f} \qquad (6b)$$

Though the form of (6) is quite complicated, it does point out that all communication between the fluid and the particle is through **T**.

$\int_S \mathbf{T} \cdot \vec{n} dS$ can be classified into a series of forces associated with discrete physical processes in the following way –

$$\int_S \mathbf{T} \cdot \vec{n} dS = \vec{D} + \vec{A} + \vec{B} + \vec{L} + \vec{M} \qquad (12)$$

Added mass

The added mass force is a result of the fluid surrounding particle being accelerated. It has a tendency to keep the particle from being accelerated in any direction.

From potential flow theory –

$$\vec{A} = -\rho c_m V_p \frac{d\vec{u}}{dt} \tag{13}$$

where c_m is the added mass coefficient. It is approximately 0.5.

Basset

A.k.a., the history term. The Basset force is the force associated with past movements of the particle. Mathematically, it takes the form (for 1D flow) –

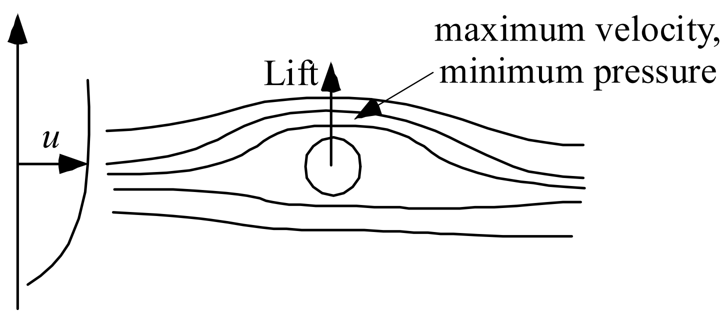
$$\vec{B} = -6\pi\mu a\left(v_p + \frac{a}{\sqrt{\pi\nu}} \int_{t_0}^{t} \frac{v'_p(t_1)}{\sqrt{t-t_1}} dt_1 \right) \tag{14}$$

where v_p is the velocity of the particle, v'_p is the acceleration of the particle, and a is the radius of the particle.

The equation up to this point (including the drag, added mass and Basset force in the stress tensor) is often referred to as the BBO equation, after Basset, Boussinesq and Oseen, who originally derived it. It is probably the most common equation to solve for the path of a particle.

Lift

Lift is one of the most important forces in moving fluid. It is a result of pressure differences around the particle.



As a result, the mathematical expression for the lift force is –

$$\vec{L} = \frac{1}{2}\rho c_L A_p \left(\left| u_r \right|_{max}{}^2 - \left| u_r \right|_{opp}{}^2 \right) \hat{k}_{max} \qquad (15)$$

where A_p is the surface area of the particle, c_L is the coefficient of lift (highly particle and flow dependent) and u_r is the velocity perpendicular to particle.

Magnus force

The Magnus force is a result of rotation of the particle. From (6), you can show that rotation of the particle will induce 'lift' (differential pressures) perpendicular to the direction of the motion (i.e., the 'curve ball effect').