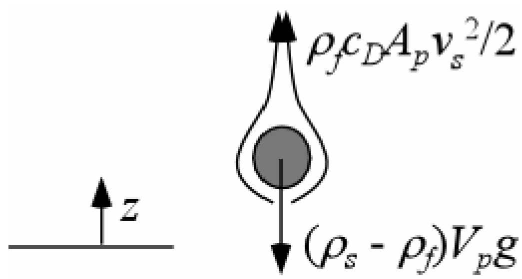
Stokes settling

Let's consider a simple force balance on a spherical particle falling through a quiescent ($u = 0$) fluid



Assuming the total drag acts through the center of mass, Drag = Gravity,

$$\frac{\rho_f c_D A_p w_s^2}{2} = \left(\rho_s - \rho_f\right)V_p g \qquad (1)$$

Knowing that $c_D = f(Re_p)$, where $Re_p = w_s D/\nu$, and rearranging (1), we find

$$\frac{w_s}{\sqrt{RgD}} = \left[\frac{4}{3c_D\left(Re_p\right)}\right]^{1/2} \qquad (2)$$

The drag coefficient's dependence on Re_p is quite complex and somewhat dependent on the shape of the particle.

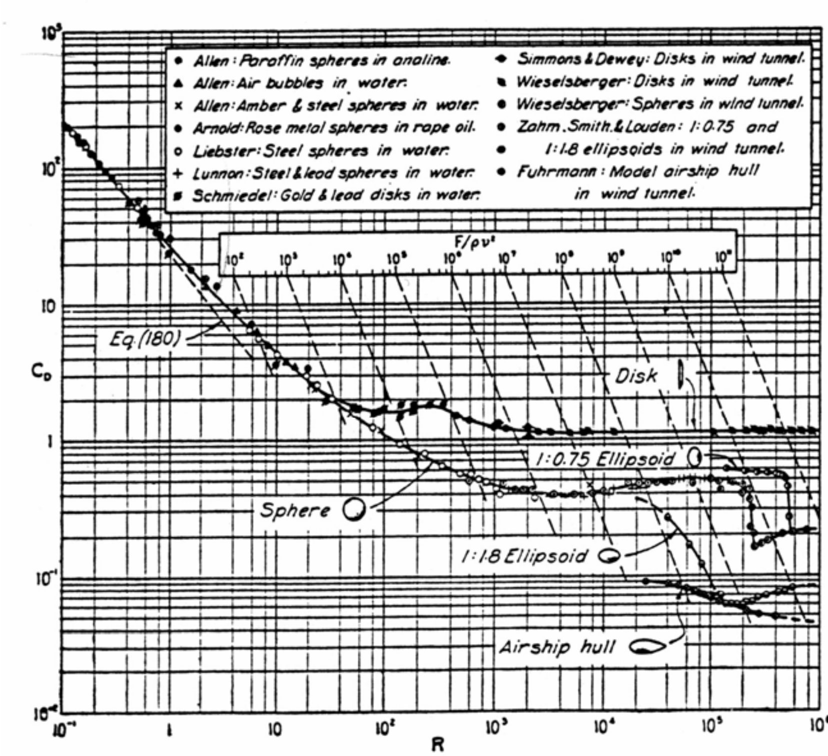


FIG. 125. Coefficients of drag as functions of the Reynolds number for bodies of revolution.

Limit of small (Stokesian) particles

As can be seen in the figure, for small Re_p, the drag coefficient is linearly proportional to Re_p. They are proportional because the flow around the particle does not separate (it is laminar). In this case, potential flow theory describes the flow field and the pressure drop related to the settling particle.

$$Drag = -3\pi\rho\nu D w_s \tag{3}$$

Drag is equivalent to the LHS of (1). Replacing (3) in (1),

$$\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 Rg = 3\pi\nu D w_s \quad (4)$$

Solving for w_s

$$w_s = \frac{1}{18}\frac{RgD^2}{\nu} \quad (5)$$

Equation (5) is the well-known **Stokes settling** equation. As can be seen Hunter Rouse's data (the figure of drag above), this is only applicable for $Re_p < 1$. In water, this corresponds to $D \sim 100$ μm. Considering that the Kolmogorov length scale $\eta \sim 100$ μm, these two results yield some insight into the strikingly different transport behavior of silt and sand in water.

Natural materials (Dietrich, 1982)

Dietrich (1982) was interested in an empirical expression that would express the settling velocity as an explicit function of particle characteristics. As we will discuss below, theoretical formulations based upon drag result in multi-valued equations.

Used dimensionless quantities –

$$W_* = \frac{w_s^{\ 3}}{Rg\nu} \tag{6}$$

$$D_* = \frac{RgD_n^{\ 3}}{\nu^2} \tag{7}$$

Where w_s is the settling velocity, ν is the kinematic viscosity and D_n is the nominal diameter of the largest projected area.

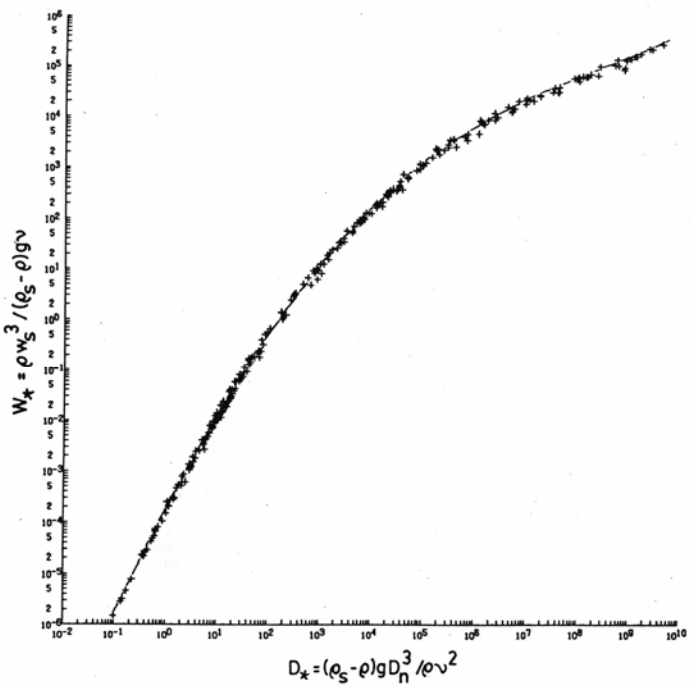
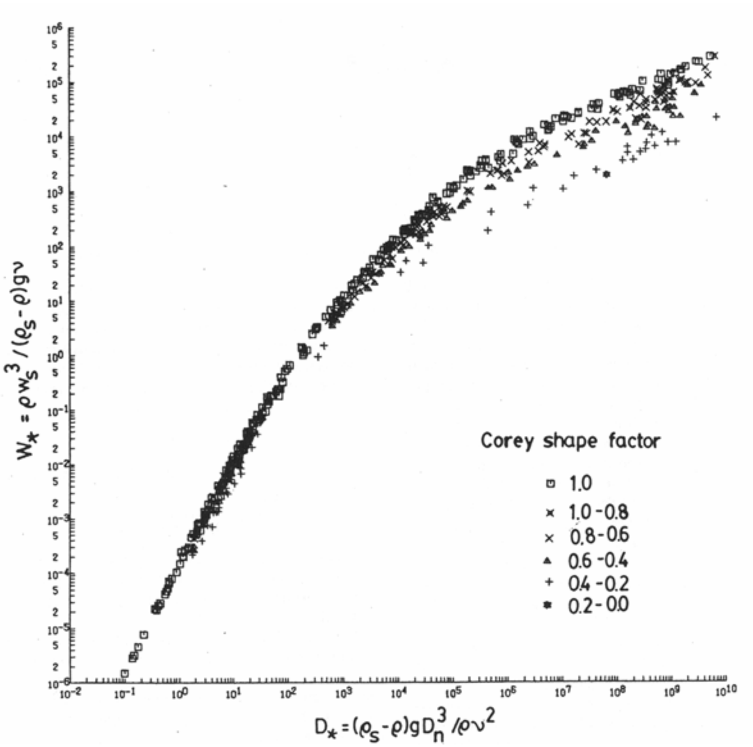


Fig. 1. Settling velocity of spheres plotted as a function of W_* and D_*. Sources of data given in Table 1. Curve is a least squares fit of a fourth order polynomial (equation (9)).

He broke up the effects of shape and its production of a turbulent wake into three coefficients in the equation –

$$W_* = R_3 10^{R_1 + R_2} \tag{8}$$

where R_1 represents the effects of 'density', R_2 represents the effects of shape, and R_3 encompasses angularity (roundness).



$$R_1 = -3.76715 + 1.92944(\log D_*) - 0.09815(\log D_*)^2 - 0.00575(\log D_*)^3 + 0.00056(\log D_*)^4 \tag{9a}$$

$$R_2 = \log\left(1 - \frac{1 - CSF}{0.85}\right) \tag{9b}$$

$$R_3 = \left[0.65 - \left(\frac{CSF}{2.83}\tanh\left(\log D_* - 4.6\right)\right)\right]^{\left(1+\left(3.5-P\right)/2.5\right)} \quad (9c)$$

where *CSF* is the Corey shape factor defined by

$$CSF = \frac{c}{\sqrt{ab}} \quad (10)$$

where *a* is the largest length scale associated with the particle, *b* is an intermediate length and *c* is the minimum length. *P* is Powers value of roundness, which is a qualitative measure of roundness described by Powers (1953). Basically P is smaller for more angular material. Perfectly round material has $P = 6$ (for which R_3 becomes equal to one). Highly angular material (crushed silica, for instance) generally has $P \sim 2\text{-}3$.

Dietrich (1982) is particularly good for fluvial and aeolian sands. Oceanic flocs require a different treatment (the 'density' effects are properly accounted for).

Particle-particle interactions

Particles interact with one another and the surrounding fluid as they settle in three principle ways:

1. *Increasing the viscosity of the fluid*. A classic result by Einstein (1905) predicts this increase for dilute suspensions –

$$\frac{\mu_{suspension}}{\mu} = 1 + k_e C \qquad (11)$$

where k_e is the Einstein coefficient (= 2.5), C is the volumetric concentration of particles, and $\mu_{suspension}$ is the total viscosity, and μ is the viscosity of the interstitial fluid. This calculation is applicable for only the "dilute limit" ($C < 1$ g/l). We will discuss what happens when the suspension is not dilute later in the course.

2. *Particle-fluid-particle interactions*. In a quiescent fluid, there are a number of interactions particles can have on one another that impede their fall. You can show that for pure Stokesian settling of an infinitely large suspension (number of particles => infinity), the impedance is divergent (i.e., infinite). Batchelor (1972) divided these into three different divergent terms and into empirical constants that were determined experimentally.

The sum of these three effects for Stokesian particles in the dilute limit is:

$$w_s = w_{Stokes}(1 - 6.55C) \qquad (12)$$

3. *Particle-particle collisions*. Particles within concentrated suspension bump into one another, slowing their settling. This is referred to as **hindered settling**. It occurs for concentrations >> 10 g/L.