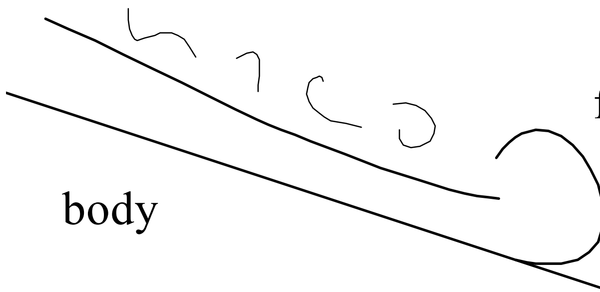
Conservative gravity currents



Investigations of gravity current dynamics have been classified into two categories –

1) Continuous
2) Lock-exchange (a.k.a., pulsed, fixed-volume)

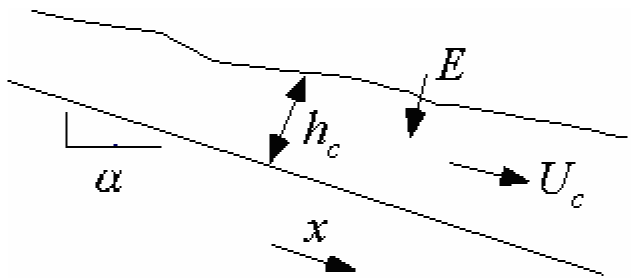
Continuous gravity currents

Ellison and Turner (1959) wanted to characterize the change in the height of the flow with distance downstream. Their key assumption is that the buoyancy flux is conserved (thus the name: conservative gravity currents)

The buoyancy flux per unit width $g'Q$ is defined as:

$$g'Q = \Delta g h_c U_c \qquad (1)$$

where Δ is the excess density difference between the two layers $(\rho_c - \rho_a)/\rho_c$, Q is the total volumetric flow rate per unit width, g' is the reduced gravitational acceleration and h_c and U_c are the height and mean velocity of the layer of interest; respectively.



However, with time and distance down the channel, fluid will be entrained. The rate of entrainment E will be related in the following way

$$E = \frac{1}{U_c}\frac{d(U_c h_c)}{dx} \tag{2}$$

From experiments (and analysis), a number of researchers have found the following empirical relationship

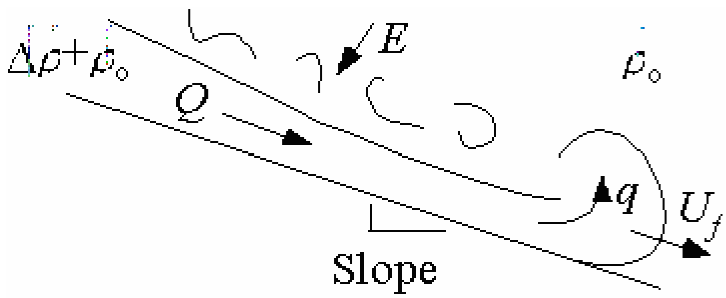
$$E = \frac{0.075}{\sqrt{1 + 715 Ri^{2.4}}} \tag{3}$$

where Ri is the Richardson number

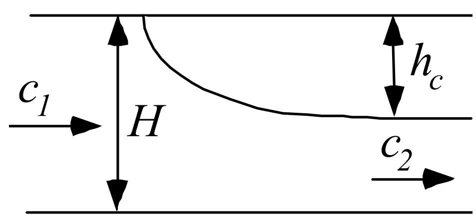
$$Ri = \frac{\Delta g U_c h_c \cos\alpha}{U_c^{\,2}(U_c + U_a)} \tag{4}$$

Lock-exchange flows and the front propagation rate

In these flows, the primary goal has been describing the propagation rate of the front.



Benjamin (1968) considered energy conservation across a water-air front



$$E_1 = \rho\left(c_1^{\,2} H + gH^2\right)/2 \tag{5}$$

$$E_2 = \rho\left(c_2^{\,2}(H - h_c) + g(H - h_c)^2/2\right) \tag{6}$$

and then continuity

$$c_1 H = c_2 (H - h_c) \tag{7}$$

Solving (5-7), we find –

$$c_2{}^2 = \frac{gH\left[H^2 - (H - h_c)^2\right]}{(H - h_c)(H + h_c)} \tag{8}$$

He later transformed reference frames and added the energy loss terms (which are zero for vanishing dimensionless depth), and found the relation

$$\frac{U_c}{\sqrt{\Delta g h_c}} = \left[\frac{(H - h_c)(2H - h_c)}{H(H + h_c)}\right]^{1/2} \tag{9}$$

This equation is only valid for $h_c/H \sim 1/3$. Flows larger than that do not conserve energy.

However, Benjamin's analysis implicitly assumed that the fluid 'mixed out' of the front is negligible. In fact, it is not. Using dimensional arguments, Huppert and Simpson (1980) pose an empirical model that incorporates mixing at the front. They assume current propagation takes place as a collapsing rectangle with the front condition of –

$$Fr_f = \frac{U_c}{\sqrt{\Delta g h_c}} = 1.19; \quad h_c/H < 0.075 \tag{10a}$$

$$Fr_f = \frac{(h_c/H)^{-1/3}}{2}; \quad 0.5 > h_c/H > 0.075 \qquad (10b)$$

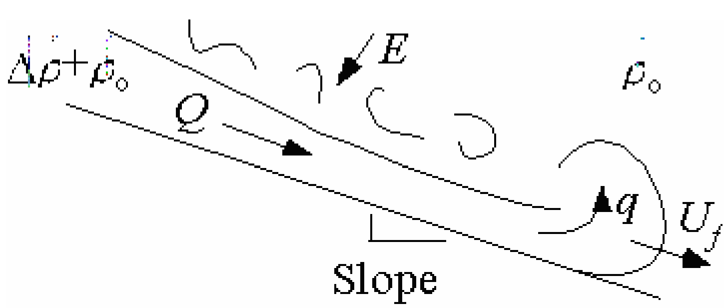
The two categories are necessary because as h_c/H increases the inertial effects modeled by Benjamin (1968) become important. For deeply submerged flows, the front propagation rate is simply a result of mixing.

Mixing at the front

Most of the mixing within a gravity current is at the front. The mixing has two major effects on the transport of dense fluid –

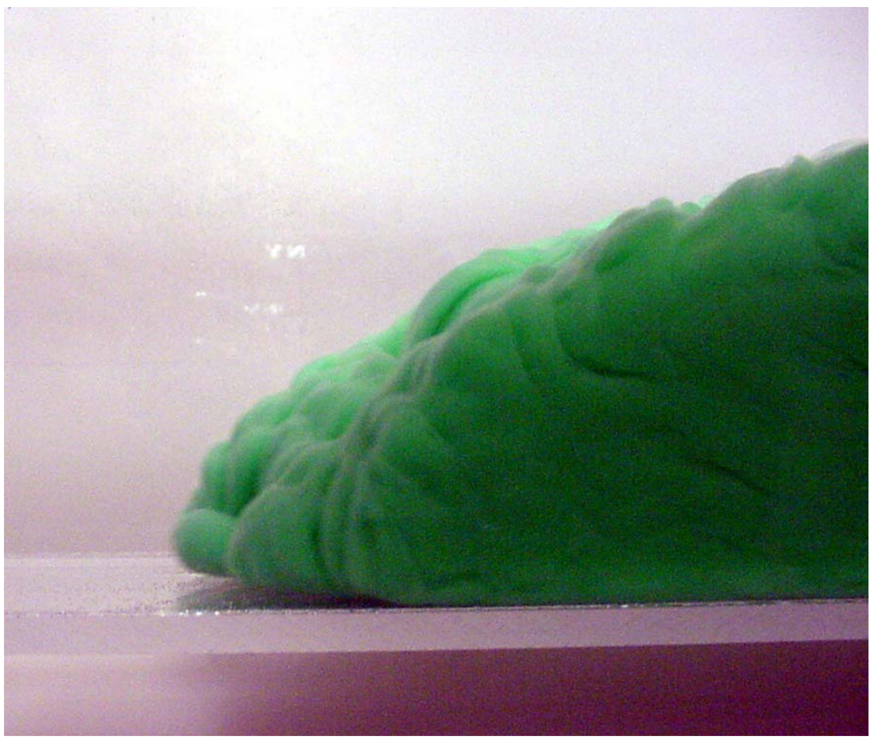
1) It locally increases the internal near-bed velocity of the flow with respect to the front propagation rate.
2) It increases the total amount of fluid transported for known current characteristics (U_f, Fr_f).

These two effects could have significant effects on the transport rate Q. If we notice –



Experiments have identified two dominant modes of mixing at the front.

1) Billowing – associated with the Kelvin-Helmholtz instability
2) Lobes and clefts - appear to 'grab' ambient fluid in a spanwise sense.



To quantify these effects, Simpson and Britter (1978) again used the concept of a Richardson number –

$$Ri_{mix} = \frac{g'q}{U_f^{\,3}} \tag{11}$$

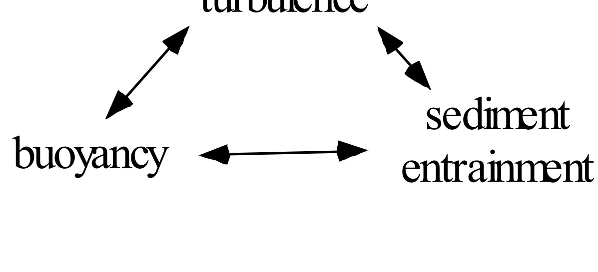
Large-scale experiments have found that $Ri_{mix} \sim 0.3$, with a slight dependence on the dimensionless depth of the flow (Parsons, 1998). These experiments also showed that gravity currents must be limited to $h_c/H < 0.15$ to conserve energy.

Turbidity (non-conservative) currents

Initial work was primarily performed by geologists, interested in understanding turbidites.

Keunen (1965) and Middleton (1966) were able to make several statements about turbidity currents.

- Turbidity currents are capable of suspending sand (early research thought this was impossible).
- Sedimentary structures found in turbidite deposits could be related to particular structures and flow regimes found in turbidity currents.
- Fine material (the fine-end of the grain-size distribution of the flow) was one of the most important variables in defining the **competency** (the ability to move material) of the flow.
- In short, turbidity currents set up a cycle –



Ignition (Autosuspension)

One of the fundamental differences between turbidity currents and subaerial water flows is that the driving force is coupled to the flow. In other words, let's assume a simple model –

$$\frac{du}{dt} = C - u^2 \tag{12a}$$

$$\frac{dC}{dt} = u^m - C \tag{12b}$$

Note that in subaerial water flows the *C* in (12a) will be replaced by some constant reflective of the water depth.

We can see that if $u = C = 1$, that the solution of this system is stable. Physically, this corresponds to an equilibrium state (erosion maintains a concentration just enough to sustain the flow). However, let's observe what happens when we perturb the system.

We will perturb *u* and *C* such that the perturbations are

$$u = 1 - u' ; \quad C = 1 - C'$$

We now examine the change in each of the perturbations with time (and neglecting nonlinear terms) –

$$\frac{du'}{dt} = C' - 2u' \tag{13a}$$

$$\frac{dC'}{dt} = mu' - C \tag{13b}$$

And if we assume that perturbations are of exponential form (waveforms of some extraction) –

$$(u', C') = (\beta, \delta)\exp(\alpha t) \tag{14}$$

We can solve (13) by using characteristics. The characteristic equation becomes

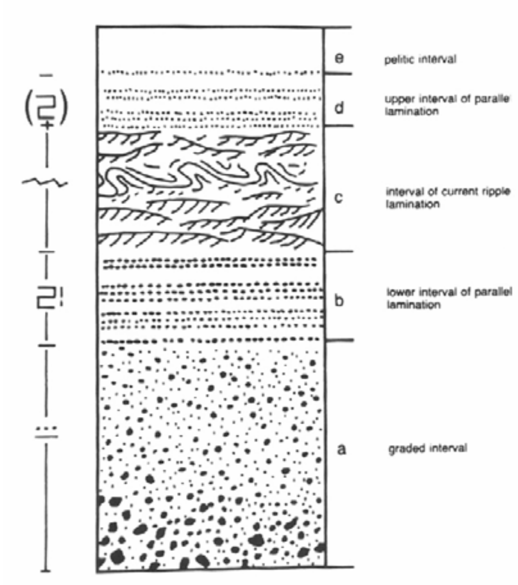
$$\alpha = \frac{-3 \pm \sqrt{9 + 4(m-2)}}{2} \tag{15}$$

But we know that $m \sim 5$. Therefore, this system is unstable and will lead to exponentially large u and C.

Parker (1982) referred to these flows as **ignitive**.

Turbidites

Bouma (1962) set about describing a series of beds in the Pyrenees. To ease interpretation of beds that he believed originated from turbidity currents, he devised a template.



Classic Bouma sequences (where all portions are present) have a tendency to be small (no more than 1 m thick in total). It is essentially a description of a waning, poorly-sorted, turbulent flow.