Longshore current production from waves

Longshore currents are common features on most coasts. They are due to three primary factors: 1) obliquely breaking waves, 2) longshore variations in set-up and 3) large-scale geostrophic motions. Radiation-stress theory can describe the first type of motion by considering that

$$S_{xy} = En \sin\alpha \cos\alpha \qquad (1)$$

where α is the angle of incidence of the wave crests.

The original analysis of Longuet-Higgins (1970) uses the equivalent form

$$S_{xy} = ECn \cos\alpha (\sin\alpha / C) = P \cos\alpha (\sin\alpha / C) \qquad (2)$$

The first term relates to the wave energy flux per unit length of shoreline, while the second term on the RHS is constant according to Snell's Law of refraction.

Onshore variability in the first term above drives the longshore current. Therefore, we must differentiate (1) with respect to x

Remember, from the previous lecture

$$\frac{\partial \bar{\eta}}{\partial x} = \left[\frac{1}{1 + 8/3\gamma^2} \right] \frac{\partial h}{\partial x} \qquad (3)$$

$$\frac{\partial S_{xy}}{\partial x} = \frac{\sin\alpha}{C}\frac{\partial\left(PC_g\right)\cos\alpha}{\partial x} \quad (5)$$

From linear wave theory, we know

$$E = \frac{\rho g H^2}{8} \quad (6)$$

Assuming shallow water waves (i.e., $C = C_g = \sqrt{gh}$),

$$\frac{\partial S_{xy}}{\partial x} = \frac{5}{16}\varsigma\rho g\gamma^2\left(\bar{\eta} + h\right)\frac{dh}{dx}\sin\alpha\cos\alpha \quad (7)$$

where $\frac{\partial h}{\partial x} = -\tan\beta$ is the slope (assumed constant) and

$$\varsigma = \frac{1}{1 + 3\gamma^2/8} \quad (8)$$

Komar presents the alternative where

$$\frac{\partial S_{xy}}{\partial x} = -\frac{5}{4}\varsigma\rho {u_m}^2\tan\beta\sin\alpha\cos\alpha \quad (9)$$

where u_m is the maximum water velocity was obtained from the shallow-water approximation.

In the original derivation by Longuet-Higgins (1970), he assumed that thrust described in would be exactly matched by frictional losses of the form a drag equation

$$\frac{\partial S_{xy}}{\partial x} = C_f \rho |u| v_l \tag{10}$$

where v_l is longshore current velocity. The magnitude of the velocity is related to wave motions, where

$$|u| = \frac{2u_{\max}}{\pi} \tag{11}$$

Substituting (10) and (11) into the LHS of (9),

$$\frac{2C_f u_{\max} \rho v_l}{\pi} = \frac{5}{4} \rho u_{\max}{}^2 \tan\beta \sin\alpha \cos\alpha \tag{12}$$

which can be manipulated into the form

$$v_l = \frac{5\pi}{8C_f} u_{\max} \varsigma \tan\beta \sin\alpha \cos\alpha \tag{13a}$$

or

$$v_l = \frac{5\pi\varsigma \tan\beta}{8C_f}\sqrt{gh_b}\sin\alpha\cos\alpha \qquad (13b)$$

which is the general form of

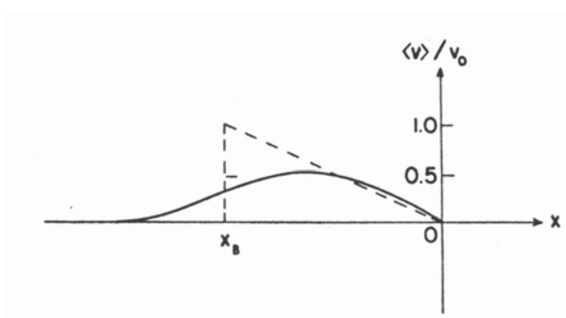
$$v_l = A\tan\beta\sqrt{gH_b}\sin\alpha\cos\alpha \qquad (14)$$

where $A = 41.4$ in the case of the *Shore Protection Manual*.

As Komar points out, the assumption of linear wave theory in and near the surf zone is a bad assumption. Ironically, the theory works reasonably well (at least to first order).

In fact, the dependence on the slope is relatively small and the entire leading term in (13b) can be taken to be ~ 1.

Cross-shore variability of longshore current



In deriving (13), Longuet-Higgins (1970) also postulated the shape of the cross-shore current profile.

The form the equations end up taking is

$$V = \begin{cases} B_1 X^{p_1} + AX \\ B_2 X^{p_2} \end{cases} \tag{15}$$

where $X = x/X_b$, $V = v/v_0$

$$v_0 = \frac{5\pi\gamma\varsigma^2 \tan\beta}{16C_f}\sqrt{gH_b}\sin\alpha\cos\alpha$$

$$p_1 = -\frac{3}{4} + \sqrt{\frac{9}{16} + \frac{1}{\varsigma P}}, \qquad p_2 = -\frac{3}{4} - \sqrt{\frac{9}{16} + \frac{1}{\varsigma P}}$$

$$P = \frac{\pi N \tan\beta}{\gamma C_f}$$

$$A = \frac{1}{[1 - (5\varsigma P/2)]}, \qquad B_1 = \frac{p_2 - 1}{p_1 - p_2}A, \qquad B = \frac{1}{[1 - (5\varsigma P/2)]}$$

These equations work well for breaking angles $\alpha < 45°$.

The mixing described by Longuet-Higgins (1970) does occur, mostly a result of the **Kelvin-Helmholtz** or shear instability.

These waves could help maintain periodic shoreline features that are of longer wavelength than traditional edge waves (Oltman-Shay et al., 1989). These waves are called **shear waves**, or sometimes **far-infragravity (FIG)** waves.

The addition of longshore variability in wave setup

Komar's PhD thesis investigated the interaction of cusps and the development of nearshore circulation cells. In doing this, he formulated the expression

$$v_l = 1.17\sqrt{gH_b}\sin\alpha_b\cos\alpha_b - a\sqrt{gH_b}\frac{\partial H_b}{\partial y} \quad (16)$$

where

$$a = \frac{\pi\sqrt{2}}{C_f \gamma^{5/2}}\left(1+\frac{\gamma^2}{8}\right)$$

From his experiments and analysis, he hypothesized that in 'equilibrium' beaches the first and second term on the RHS of (16) cancel each other out.

However, many beaches are not in ‘equilibrium’. In this case, transport longshore will be enhanced by the energy distribution of the imposed wave field.