Propagation and transformation

Komar suggests four mechanisms for wave attenuation

1. Angular spreading

2. Contrary winds

3. Nonlinear wave-wave interactions

4. Internal viscous damping

Ultimately, the energy all must be converted to heat via viscous damping.

Keulegan (1950), who presents an outstanding discussion of wave motions, suggests

$$H = H_i \exp\left(-\frac{8\pi^2 \nu t}{{L_\infty}^2} \right) \quad (1)$$

where H_i is the height of waves of the *i*th wavelength and $L_\infty = gT^2 / 2\pi$, so

$$H = H_i \exp\left(-\frac{32\pi^4 \nu t}{g^2 T^4} \right) \tag{2}$$

You can describe (1) and (2) in terms of a half-life $t_{1/2}$, which describes the time required for the amplitude (height) of the wave to become half its original value.

$$t_{1/2} = 0.0088 \frac{L_\infty{}^2}{\nu} = \frac{0.0088}{4\pi^2} \frac{g^2 T^4}{\nu} \tag{3}$$

Keulegan, G. H. (1950) Wave motion. In: *Engineering Hydraulics*, ed. Rouse, H. John Wiley & Sons.

Shoaling

There is a large amount written (and many theories published) about shoaling. However, there are very few simple theories that do not require significant knowledge about the wave spectrum and bathymetry. The simplest is Rayleigh (1911), where he showed that, if Airy theory holds –

$$\frac{L}{L_\infty} = \sqrt{\tanh\left(2\pi h / L_\infty\right)} \tag{4}$$

If you remember, this is the recommended way the *Shore Protection Manual* suggested you calculated L in intermediate depths.

If you plot C_g vs. depth (Komar's Fig. 5-21), you'll notice that the group velocity actually increases before it drops in the nearshore.

A way to calculate what happens to the wave height is to assume that energy is conserved (i.e., no energy is lost to friction). That is,

$$P = EC_g = \left(EC_g\right)_\infty = \text{Constant} \tag{5}$$

which results in the expression

$$\frac{H}{H_\infty} = \sqrt{\frac{1}{2n}\frac{C_\infty}{C}} \tag{6}$$

Remember

$$n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right] \tag{7}$$

for Airy waves.

In actuality, P is consumed by friction with the bottom. Considering that all of the common wave theories (Airy, Stokes and cnoidal) use an inviscid approximation, we need some *ad hoc* way of including the effects.

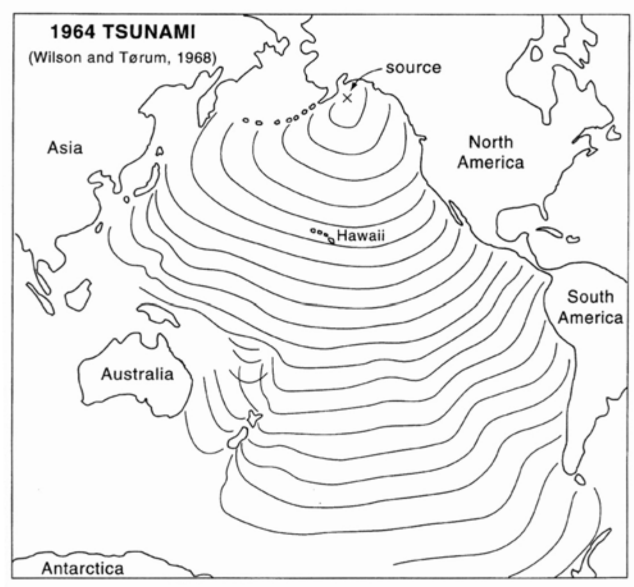
Madsen et al. (1988) propose a wave friction factor f_w, where

$$\frac{dP}{dx} = -\frac{1}{16}\rho f_w u_0^3 \tag{8}$$

where u_0 is the wave orbital velocity at the bottom and x is in the onshore direction. Madsen et al. (1988) postulated $f_w \sim 0.1$, however, it vary WIDELY. As a result, as Komar suggests, in most applications dP/dx is ignored.

Refraction

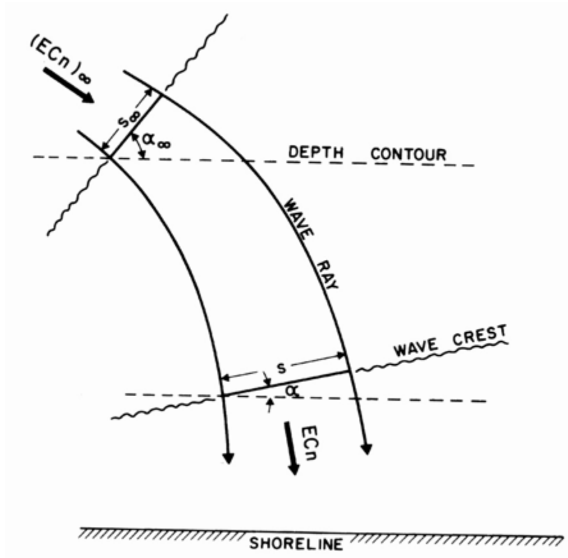
Not only do waves become damped as they propagate, but the bottom can also alter their direction of motion. Primarily waves are altered because of refraction.



For slowly varying depths (i.e., $\tan\beta << kh$ – as is usually the case on sandy, passive margins), Snell's law of refraction applies:

$$\frac{\sin\alpha_1}{C_1} = \frac{\sin\alpha_2}{C_2} \qquad (9)$$

The waves are not only bent, but spread along the coast. As a result, some energy is lost.

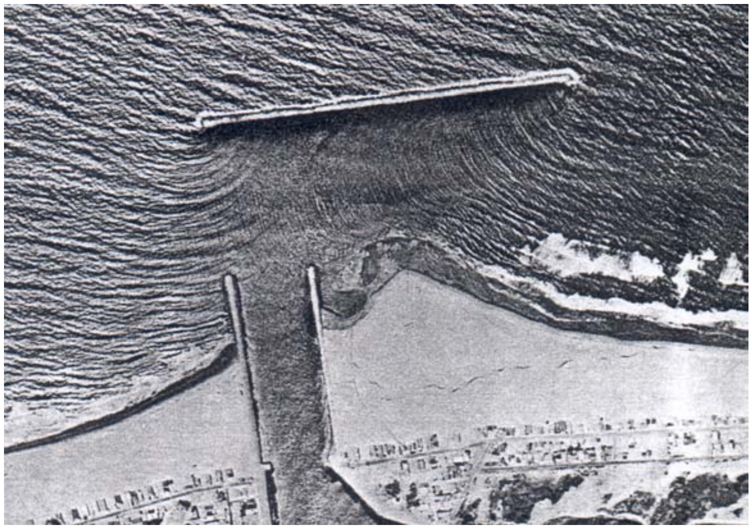


Using Equation (6) and assuming that energy is not being dissipated, we find

$$\frac{H}{H_\infty} = \sqrt{\frac{1}{2n}\frac{C_\infty}{C}\frac{s_\infty}{s}} \qquad (10)$$

The last term in this equation $K_r = \sqrt{s_\infty / s}$ is often called the refraction coefficient. For a straight coast with parallel isobaths, $K_r = \sqrt{\cos\alpha_\infty / \cos\alpha}$.

Diffraction



Channel Islands harbor, California
(from Army Corps Shore Protection Manual, 1984)

Refraction, diffraction and shoaling are now commonly managed by large numerical codes. For instance, the WAM model (Komen et al., 1994) is capable of doing this.

Breaking waves

Breaking is one of the best manifestations of the presence of non-linear effects in water waves.

Waves can be broken in deep water. Here, the breaking criterion is

$$\frac{H_\infty}{L_\infty} \approx \frac{1}{7} \tag{11}$$

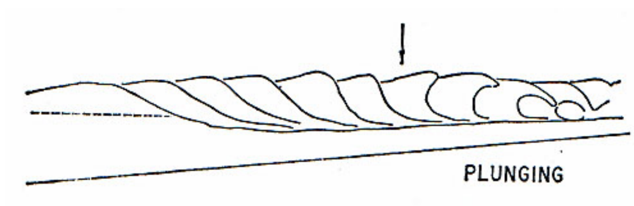
where before they break, they move like Stokes waves.

Breaking at locales near shore gives rise to the **surf zone**. Surf-zone breakers are shallow, so they move as $C = \sqrt{gh}$. They also result more from bottom topography.

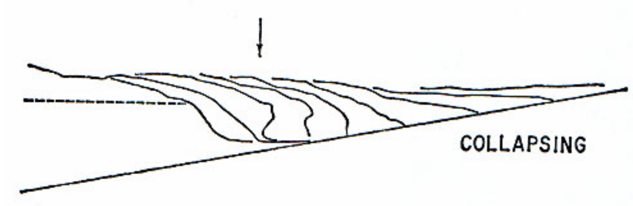
The nonlinearity of the process makes most analysis futile – so most of the work has been done in the lab. These have classified waves into four categories:

Spilling

Plunging



Collapsing



Surging

Figures obtained from Komar

Battjes (1974) used a variable $\xi_\infty = \sqrt{\frac{H_\infty}{L_\infty}}\tan\beta$, where H_∞ and L_∞ are the deep-water wave height and length and β is the beach slope. The variable ξ is sometimes called the Iribarren number. His experiments suggested –

Spilling: $\xi_\infty < 0.5$
Plunging: $0.5 < \xi_\infty < 3.3$
Collapsing-Surging: $\xi_\infty > 3.3$

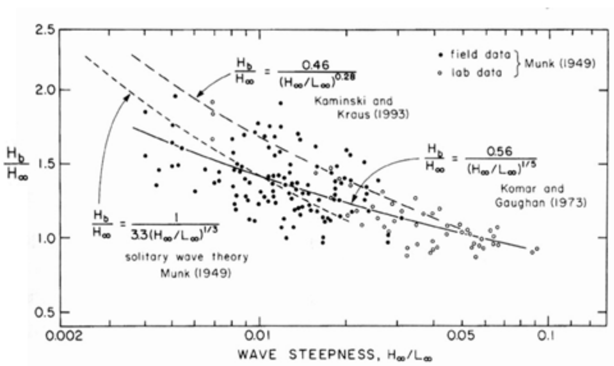
The breaker height can (and has been) cast in terms of deep-water parameters. One popular model is Komar and Gaughan (1972). They suggest

$$H_b = 0.39 g^{0.2} \left(T H_\infty{}^2 \right)^{0.4} \tag{12}$$

However, the most appropriate form to compare is the dimensionless form

$$\frac{H_b}{H_\infty} = a \left(\frac{L_\infty}{H_\infty} \right)^b \tag{13}$$

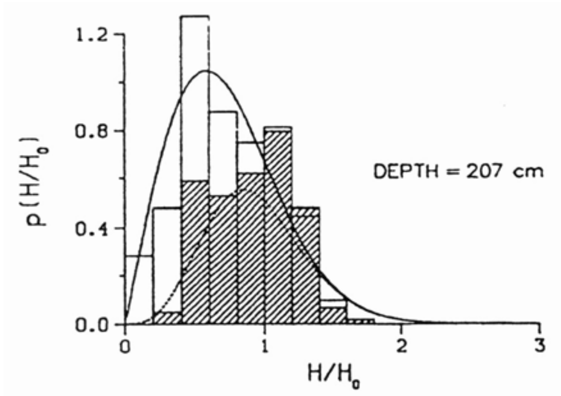
where *a* and *b* are constants to be determined. A wide range of data has been collected, suggesting $0.3 < a < 0.5$ and $0.2 < b < 0.3$.



Best surfing conditions????

Wave decay after breaking

Unlike open-ocean waves, waves in the surf zone behave slightly differently than a Rayleigh distribution.



Nonlinear interactions between waves of different frequencies cause some ways to be produced at the expense of others.

Individual waves in surf zones are impossible to predict, so stochastic models rule the day. Thorton and Guza (1983) propose

$$p(\eta)=\left(\frac{H_{rms}}{\gamma h}\right)^{n}\left\{1-\exp\left[-\left(\frac{\eta}{\gamma h}\right)^{2}\right]\right\} \tag{14}$$

where γ is the depth-limiting condition on the wave height. Thorton and Guza (1983) propose

$$\gamma=\left(\frac{H_{rms}}{h}\right)_{at\ breaking}=0.42 \tag{15}$$

which is also consistent with an old Gidget episode I once saw.

It's also interesting to note that the 'saturated' state of the flow – that is, the point at which all waves will be roughly equivalent to

Equation (15). From Russell's original result for solitary waves, we find

$$C = \sqrt{g(h + H)} = \sqrt{gh(1 + H/h)} = 1.19\sqrt{gh} \tag{16}$$

This is IDENTICAL to the empirical result of Huppert and Simpson (1980) for propagation rates of gravity currents. Neither paper was apparently aware of the other.

To consider the bulk effect on the dynamics (and ultimately the mean water depth), we need to understand the rate at which energy is being dissipated due to the breakers. Mathematically, this is

$$\frac{\partial(ECn)}{\partial x} = -\varepsilon(x) \tag{17}$$

where $\varepsilon(x)$ is the loss in wave energy per unit area per unit time.

An example is from Thorton and Guza (1983), who used the formulation

$$\varepsilon = \frac{\rho g f (BH)^3}{4h} \tag{18}$$

where f is the frequency of the waves and B is the breaker coefficient (Komar says, $B \sim 1$).

There are many surf-zone models out there, which generally have the form of Equation (17). Komar presents a good discussion of these models, from which you could find one appropriate to your environment.