Notes on SOLAR RADIATION and the EARTH’S GREENHOUSE

Sun’s radiation and its spectrum. We introduced the sun’s energy, and the spectrum of wavelengths that describe the radiation. The color of the light is related to its wavelength, and outside of the band of visible colors (0.39 – 0.78 µm, 10^-6 m, that is millionths of a meter). Light has many properties of a wave, an electromagnetic wave, particularly its wavelength $\lambda$ (m), frequency $f$ (Hertz, or cycles per second) and wave speed, c (m/sec). These are related in the usual sense that distance from one wave-peak to the next $= \text{speed} \times \text{time}$ it takes a wave-peak to recur, which translates here to wavelength $= \text{speed/frequency}$: $\lambda = c/f$. The speed of light $c = 3 \times 10^8$ meters/sec in a vacuum, but it is slower in glass or plastic or other transparent media. Hence we have refraction, which bends light rays as they move, say, from air to glass, giving us lenses and prisms. The speed $c$ varies with wavelength, so that white light, which combines all colors, is split into its rainbow of component waves. Of course it is very fast, light taking 8 minutes 20 sec to travel the 150 million km from the sun. But physicists have succeeded in slowing light to a walking speed in the lab!

Figure: color of light as a function of its wavelength, in nanometers ($10^{-9}$m, billionths of a m.).

We also described several triumphs of 1800s physics:

1) Maxwell’s equations describing electromagnetic (e.m.) waves and unifying much of electricity, magnetism. They show how a changing electric field (like a moving electron) produces a magnetic field and a changing magnetic field produces an electric field. This is the basis for both electric motors and e.m. waves.

The origin of electromagnetic waves is the vibration of charged particles making up atoms and molecules. When electrons orbiting about the nucleus jump from one energy level to the next they can absorb or emit e.m. radiation. More complex oscillations of the electron-proton (minus and plus) charges occur. A radio antenna, which might be 40m long to match with the wavelength of a 7 megaHertz (7 million cycle per second) wave is an orderly pattern of oscillating electrons in a conducting wire.

2) Planck’s law showing the shape of the spectrum (that is, the distribution of radiating energy among different wavelengths) of radiation from a hot planet, assumed a ‘perfect black
body’ (that is, a body that radiates perfectly without any radiation being absorbed on the way out): If \( B(\lambda) \) is the radiated power, as a function of wavelength

\[
B(\lambda) = \frac{2hc^2}{\lambda^5} \exp\left(\frac{hc}{\lambda kT}\right) - 1
\]

where \( h \) is known as Planck’s constant, \( k \) is known as Boltzmann’s constant, and \( c \) is the speed of light.

What are the physical units of \( B \)? It is power (in watts) over meters\(^3\). The total area under the

\[\int B(\lambda) \, d\lambda\]

is the Stefan-Boltzmann law,

\[
F = \sigma T^4
\]

\( \sigma \) is called the emissivity and is a fundamental constant, \( 5.67 \times 10^{-8} \text{ watts/(m}^2\text{ K}^4) \), for an ideal ‘black body’ that radiates perfectly; in reality it varies somewhat with wavelength.

The units of \( \sigma \) are complicated but just think of them as being whatever is needed to turn \( T^4 \) into watts/meter\(^2\) of \( F \). Again, this is for a somewhat idealized body, whereas actual radiation can have a dependence on wavelength. For the sun it turns out to be very accurate except for some absorption as the radiation passes outward from the interior, at special wavelengths related to the chemical elements present.

These formulas, particularly 3) and 4), give us the remarkable (I would say amazing) ability to know the temperature of the visible sun, and the power it puts out. The solar spectrum peaks near a wavelength of 0.5 µm and using Wien’s formula, this corresponds to a temperature
of 5800°K. However Earth has a mean temperature of 290°K and this corresponds to much longer wavelength of 10 μm…10 microns. We shall see how this difference in wavelengths, incoming radiation versus outgoing radiation, leads to the greenhouse effect and the ‘warm’ Earth. The Stephan-Boltzmann law 4) tells us that at this temperature, and knowing the area of the sun’s surface, the total energy radiating from the sun can be calculated. So the sun is bright, huge (contains 99.8% of the mass of the solar system and its radius is 109 times that of Earth), old (4.6 billion years) and nuclear fusion in its high-pressure, high temperature interior releases huge amounts of energy.

The solar spectrum observed at the ground (averaging over many locations and times) is just below.

![Solar Spectrum](image)

The visible light band is nearly one-half of the power in the radiation, and infrared between 750 and 2500 nanometers (0.75 and 2.5 microns) is about one half of the total radiation. The small amount of ultraviolet short wave energy is nonetheless very important, and the ozone shield protects us from receiving more.

This figure shows a complicated up-and-down curve, yet in outer space it is much simpler. The role of the atmosphere in this figure is clear if we look at next figure below, the sun’s radiation just above the atmosphere, the smooth, upper solid curve in near the top of the figure. The dashed, smooth curve close to it is the theoretical prediction from Planck’s law, which is very close to the observed spectrum. This curve is drawn by choosing the temperature T of the radiating object (the sun) so that the area under the curve is equal to the observed radiation. Once this is done, Wien’s law tells us the temperature of the sun! That is the parts that do the radiating. That number is 5900 K, very hot indeed. The lower solid curve is the observed incoming radiation at the ground, as above.
after blockage by molecules of air have captured some of the spectrum (actually the ‘smooth solid
curve’ has some fine wiggles which are not shown, owing to this absorption within the sun). Molecules vibrate at certain natural frequencies, and this intercepts radiation of the same frequency, and re-radiates it at other frequencies.

Notice that the peak of the sun’s radiation is in the visible light band. This is likely not a coincidence! We have evolved with eyes that see better than eyes sensitive to weaker radiation at other wavelengths...although some creatures benefit from seeing infrared ‘heat’ radiation as they stalk their prey. Notice also that ozone...O₃...absorbs much radiation in visible and ultraviolet (UV) wavelengths. This is ‘good’ ozone, which is located in the middle levels of the atmosphere (the lower stratosphere, to be exact) and protects us from UV radiation. UV can be very destructive to human skin and to the oceanic plankton that are crucial in producing oxygen for the atmosphere. Depletion of ozone has occurred in the past 2 decades by human use of CFCs (chlorofluorocarbons...like Freon refrigerants and spray can propellants) occurred over the Antarctic particularly, but also less severely over much of the planet. CFCs have been banned by the Montreal Protocol, but are still used in some countries.

The broad foothills of this spectral mountain on the right are the infrared---longer waves that we cannot see but we feel as heat radiation. This is the range 1. to 10. microns roughly. To sense infrared radiation, hold a silvered bowl close to your face. It will act as a heat mirror and you will feel the radiation from your cheeks bouncing back at you. Digital radiation thermometers (for sale by Radio Shack for example) sense these long waves and give you a fairly
accurate temperature reading of the object you are pointing at (though our formulas above show how relating radiation to temperature requires a rather sophisticated instrument). These are indeed waves: infrared cameras exist (‘night scopes’ used by the military). If you look at satellite images of weather (which I recommend; UW has a premier website with animated weather data…www.atmos.washington.edu under weather and climate) often you will see infrared images, because they see the weather whether it is night or day…they are seeing temperature rather than reflected sunlight.

<table>
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<th>Name</th>
<th>λ (Angstroms)</th>
<th>Origin</th>
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<tr>
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<td>terrestrial H₂O</td>
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<tr>
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<td>C</td>
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<td>Mg I</td>
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<td>F</td>
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<tr>
<td>G</td>
<td>4300</td>
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<tr>
<td>H</td>
<td>3968</td>
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The above figure shows the work of Joseph Fraunhofer who by 1815 had mapped more than 300 absorption lines in the sunlight reaching Earth. These are visible with a simple spectroscope. The table shows the molecules responsible for some of these lines. www.aal.lu/SPECIAL_TOPIC/6/

**How warm is Earth?** A first calculation (which can be seen in Harte’s Spherical Cow book, p69) is as follows. The sun’s radiation amounts to about 1372 watts/m² just above the atmosphere (it varies with the season because of the small change in distance from the sun, and with the 11 year solar sunspot cycle). Some of the Earth is in nighttime, some in day, some in between. The surface area is $4\pi r^2$ where $r$ is Earth’s radius; yet what intercepts the sunlight is the ‘disk’ of the Earth, that is the circle of area $\pi r^2$. If we call 1372 by the symbol $\Omega$, so the radiation averaged over the entire Earth is just $\frac{1}{4}\Omega$. Write the incoming radiation as $F_{in} = \frac{1}{4}\Omega = 368$ watts/m² with units of watts…power. We then want to forget about the part of this that is simply reflected, mirror-like, back to space. A fraction $a$ (for albedo) is reflected and on average, $a \approx 0.3$. White snow is very reflective ($a \approx 0.9$) but the deep blue sea is not ($a \approx 0.2$ or less). The oceans cover 70% of the Earth so the average albedo is small. We then have the remaining radiation, $(1-a)\Omega/4$

being absorbed by the oceans, land and atmosphere. But, averaged over a year, we don’t find the Earth gaining or losing large amounts of heat. So what comes in must go out again. The warm
Earth radiates 10 µm infrared energy, corresponding to its surface temperature. Call this outgoing heat radiation $F_{\text{out}}$ (in watts/m²). It is from (4), above,

$$F_{\text{out}} = \sigma T_0^4$$

where $T_0$ is the Earth’s surface temperature. Our heat balance for the Earth is $F_{\text{in}} = F_{\text{out}}$ or

$$(1-a)\Omega/4 = \sigma T_0^4$$

or

$$T_0^4 = (1-a)\Omega/4\sigma.$$  

Putting in the numbers, $T_0 = 255$ K or $-180^\circ$C or close to $00^\circ$ F.

The ‘GoreTex blanket’ of the greenhouse. This is too cold. It might be a good theory for the temperature of the moon, which has no atmosphere. At the level of a Google search that temperature is 127°C (day) and -173°C (night), which averages to -23°C. Not too far from -18°C (our prediction) but the albedo $a$ is quite small for grey rocks and dust of the moon...likely less than the 0.3 value used above. At this temperature the entire Earth would be frozen. Indeed there have been times far in the past when the oceans seem to have frozen over (called ‘snowball Earth’). But not lately. This is the temperature that an Earth satellite, painted black, would average as it orbits in full view of the Sun. The atmospheric temperature decreases strongly as you go upward. A few km up, above most of the mass of air, the temperature 255K can be found, and there this calculation ‘works’.

But below that altitude we have a blanket keeping us warm. Analogous to the idea of the fabric GoreTex which lets air through but blocks moisture, it lets the visible sunlight through but blocks the infrared upward radiation from the warm Earth. The average temperature of the surface of the Earth is about 290K ($+17^\circ$C), about 35°C ($63^\circ$F) greater than this radiation model suggests!

The idea arose with Joseph Fourier (1824) and the role of carbon dioxide (and a theory of ice ages) with Svante Arrhenius (1896), e.g. [http://en.wikipedia.org/wiki/Greenhouse_effect](http://en.wikipedia.org/wiki/Greenhouse_effect) It is discussed in McKibben’s book, *The End of Nature*.

Water, the H₂O molecule, causes about 60% of the greenhouse effect; carbon dioxide, CO₂ (26%). Methane, CH₄, ozone, O₃ and nitrous oxide, N₂O also are important. Carbon dioxide is present only at a level of 380 ppm (parts per million, 10⁻⁶), that is 0.038% of air. Yet it and the other greenhouse gases have an enormous effect. Because water vapor depends very sensitively on atmospheric circulation and ocean sources of water, this part of the ‘greenhouse’ is difficult to predict.

Let us make a simple model of the ‘greenhouse’, by saying that the upward infra-red radiation is blocked and absorbed by a layer of atmosphere, as it would be by a pane of glass. This also appears in Harte’s book, p160.
The diagram below shows the situation. At first it seems hopelessly complicated.

When the infrared waves are absorbed by a thin layer of atmosphere, they heat it up and it again radiates according to the Stephan-Boltzmann law. This new radiation goes both upward and downward...that is the key. It goes in all directions. So it will re-heat the ground below, and that will change the upward radiation from the ground. Complicated. But, realize that for steady conditions we must have just the same radiation upward and downward in the lower atmosphere. Write for the downcoming visible sunlight

\[ I = (1-a) \frac{\Omega}{4} \]

Now above the greenhouse lid, the upward radiation must equal I. This is the radiation from the hot atmosphere layer. Its downward radiation is the same, I again. So, in the lower atmosphere we have a total downward radiation of

\[ 2I \]

that is, the sunlight plus the re-radiated infrared coming down from the greenhouse lid. Thus we think of the blanket. For this to be in steady state, an equal amount, 2I must be radiating upward. This is

\[ \sigma T^4 = \text{and this gives us our answer: the greenhouse effect increases Earth's surface temperature by the factor } 2^{1/4} \text{ or } 1.19. \]

This sounds like a small change but remember it multiplies the absolute, Kelvin temperature. We find that the greenhouse trapping of radiation gives a new temperature of

\[ 1.19 \times 255 \]

or 303°K, or 33° Celsius. Too hot, in comparison with the observed Earth! Before discussing why, let's go one more step. We know the atmosphere is not a single pane of glass. Suppose it is more like a stack of thinner panes of glass with air in between each one. The above calculation predicts the temperature at the upper most layer, and if we continue downward, the next layer has downward infrared radiation of 2I watts, plus the sunshine amounting to I watts. The sum, 3I is now what determines the temperature of the 2d layer down, from

\[ \sigma T^4 = 3I. \]

Hence the temperature is increased by a factor 3^{1/4} or 1.32 above the original estimate. The n\textsuperscript{th} layer reaches a temperature of \( (n+1)^{1/4} T_0 \). Clearly it is getting hotter, the thicker the blanketing atmosphere. This is why the surface temperature of Venus is 750°K!

Venus has a CO\textsubscript{2} atmosphere, which effectively blocks the IR, and it is thick.

The single-pane of glass greenhouse model above made the Earth too hot.
The atmosphere is like a ‘distributed’ pane of glass and does not totally absorb the upward moving infrared heat waves as we assumed in the simple model. This can be seen on the accompanying slides (another Acrobat .pdf file) you will see more detail about solar radiation, in particular the fraction of the infrared that is absorbed and re-emitted by the atmosphere. You will also see in the figures the Earth’s orbit that creates the seasons. The tilt of the Earth’s axis (axis about which it rotates) is currently 23.5°, which makes the summer and winter contrast as we in the northern hemisphere are tilted more toward the sun, and have longer days with the noontime sun sitting higher in the sky (at angle in degrees given by {90 + 23.5 - our latitude} in degrees at the summer solstice, June 21) above the horizon. In Seattle, at 47.5° latitude, this angle is 66° above the horizon yet at the winter solstice (December 21) the sun is favoring the Southern Hemisphere and our noon-time sun angle is only {90 – 23.5 – our latitude}, or 19° above the horizon…pale sun indeed. The Arctic Circle is defined as the latitude where the sun is on the horizon at ‘high noon’ on 21 December, which from the above formula is the latitude 66.5°, passing through Raufarhöfn Iceland, Sisimiut Greenland, Vikna Norway, and Fort Yukon Alaska.

Look back at the figure showing the solar radiation spectrum. If the greenhouse idea is right, why don’t we see the infrared radiation at sea level being greater than the same wavelength radiation above the atmosphere? It is because this figure shows the direct radiation seen looking at the disk of the sun, and not the diffuse radiation coming from all other directions. The downward coming, greenhouse radiation arrives at the ground from the whole sky, not just from the small disk of the sun.

The total radiation hitting the Earth’s surface looks more like:

![Solar Radiation Diagram](image)

The numbers are in watts per square meter. The top left shows the solar radiation (342 watts/m² on average) incoming, minus the reflected sunlight (the 30% albedo effect) yielding 235 watts/m². Of this most (168 watts/m²) reaches the ocean surface and land, much of it centered on 0.5 micron visible light. After being absorbed some of this heats the lower atmosphere by conduction and carried as moisture; yet much is converted to infrared radiation, some 20 times longer wavelength (10 µm), sent upward, but absorbed by the atmosphere and in part sent back to Earth. It’s complicated, but the net result is that we feel an extra 324 watts/m² of recycled infrared heat waves, on top of the 168 watts/m² of direct sunshine. This is the greenhouse effect. Note that this means that the added downward infrared radiation makes the total downward radiation 3 times that due to the visible sunshine alone. Our single-pane-of-glass model merely
doubled the downward radiation. It made a ‘too hot’ Earth because in that model we neglected the absorption of some of the visible light on its downward trip (the 67 watts/m² seen veering off in the yellow arrow at left, above).

A more complete analysis from the latest IPCC assessment of climate change (IPCC 2007) is this figure, which shows the ‘reverberating’ infrared thermal radiation trapped by the atmosphere.

The surface temperature of our neighbor towards the sun, Venus, is about 750°C, due to this effect: it has ‘too much’ atmosphere, and it is mostly carbon dioxide. Our outer neighbor Mars is -53°C because Mars has so little atmosphere. Earth is nestled in between the too-hot Venus and too-cold Mars. We have just the right atmosphere to keep water liquid and thus make life possible. But this is changing as we tamper with Earth’s chemical makeup.

*Bottom line.* There are a lot of formulas and numbers in this lecture; they are not important in detail for this course, but we want to sense the way energy flows from the sun and is concentrated, to just the right degree, not too cold or too hot, in the thin atmosphere and oceans of Earth. Begin to sense the size of these energy flows…a 100 watt light bulb is familiar enough, but remember that 100 watts is the power coming into it that you pay for, at about a penny per hour. Most of this goes into heat and about 20 watts comes out as visible light. Compact fluorescent lights now are just as bright with a 20 watt rating. Touch one…it’s not hot!

So, in the end, how much energy flows from the sun, our benefactor? We could use the Stephan Boltzmann law for energy radiation per square meter of the sun’s surface, and multiply by the area of that surface, or we could take the incoming energy flow just above the atmosphere (roughly 1372 watts/m²) and multiply by the area of the huge sphere whose radius is the distance...
to the sun. The latter method, using 150 million km for that radius, gives: $4\pi R^2 x 1372 = \text{energy flow from sun} = 3.9 \times 10^{26} \text{ watts}$. That’s the bottom line.