GLOBAL POPULATION notes Hon A&S 222a 2006 P.B.Rhines

Below are estimates of the world's population from the year 1 A.D. to 2050 A.D. Plot these on the two graphs below. Call population P and time in years t. The first is an ordinary plot. In the second we distort the vertical axis exponentially…as if we were plotting log(population) as a function of year (that's log to the base e, not log to the base 2). This has the advantage that a simple exponential curve, $P = P_0 e^{\alpha t}$ will appear as a straight line.

The slope of the line is α (the 'growth rate' and the x-intercept (the value of P where t=0) is the constant P_0. So this plot tells you immediately: if the curve is exponential, and if so, what its growth rate is. For bank interest α is the interest rate. We describe in the lecture notes how the growth rate is related to the doubling time: Take the log of both sides of the equation

$$\log(P) = \log(P_0) + \alpha t$$

Now the great thing about logarithms is that log(A)-log(B) = log(A/B) so this is just

$$\log(P/P_0) = \alpha t$$

So P is twice P_0 when

$$\log(2P_0)-\log(P_0) = \alpha t$$

$$\log(2P_0)- \log(P_0) = \log(2P_0/P)= \log(2) = \alpha t,$$

or

$$t = \log(2)/\alpha$$

Now log to the base e of 2 is just the number 0.6931 so we see that

The doubling time of an exponential process is about 69 divided by the growth rate in percent, $t_{double} = 69/\alpha$. Actually, it's easier to remember 70 than 69, so let's say the *doubling time is about 70 over the growth rate*. A ten-year mortgage at 7% annual rate, will cost you twice the value of the loan (70/7 = 10 years) if you pay it off all at the end. A 7 year mortgage at 10% interest will do the same. Very useful. A savings account with 3% interest will double your money in 70/3 years, that is 23.3 years…actually 69.31/3 which is 23.1 years. Inflation will just about cancel this out, or maybe more.

A Porsche Speedster 356a with the 1600 c.c. engine cost $2950 in the year 1956. A Porsche Boxster with some accessories now costs about $48,000. This is 4 doublings of price (3000=>6000=>12000=>24000=>48000). So the doubling time is 50 years/4 = 12.5 years. This is a 'inflation' rate of 69/12.5 or **5.5%** . This is typical, larger than the 'official' inflation rate which should be viewed with suspicion (unless you think that today's Boxster is better than a 1956 356a, which I don't).

A useful thing about semilog plots like the is that if you double a number on the vertical axis, you move a fixed distance, no matter where you start (try it). So, for example the straight line that, on this plot, is exponential growth, doubles every 69/α years no matter where you start.

P is an exponential function of t, and the converse relation is that t is a logarithmic function of P. A log curve, $t = (1/\alpha)\log(P/P_0)$. The exponential curve starts off slowly but eventually rises faster than any power of t (like, t^2, t^3,…). The log curve eventually rises more slowly than any power of t.

As you plot the points notice how the slope changes…and estimate the growth rate from this slope. Note the Black Death years in the 1500s. See the lecture notes (Lecture 11) for more on population. Regarding the slope, there is one confusing thing: the vertical axis (10^1, 10^2, 10^3…) has numbers for P, the population. It is the log of P that has to be used to measure the slope and get the growth rate α. So, those numbers could be written also on the vertical axis: $\log(10^1)$, $\log(10^2)$, $\log 10^3$... which are 2.306, 4.605, 6.908….each separated by 2.306.

year	**population (millions)**
1	170
200	190
400	190
500	190
600	200
700	207
800	220
900	226
1000	254
1100	301
1200	360
1250	400
1300	360
1400	350
1500	435
1600	545
1650	470
1700	600
1750	629
1800	813
1850	1,128
1900	1,550
1910	1,750
1920	1,860
1930	2,070
1940	2,300
1950	2 519
1955	2 757
1960	3 023
1965	3 337
1970	3 696
1975	4 073
1980	4 442
1985	4 843

1990	5 279
1995	5 692
2000	6 085
2005	6 464
2010	6 842
2015	7 219
2020	7 577
2025	7 905
2030	8 199
2035	8 463
2040	8 701
2045	8 907
2050	9 075

Sources:

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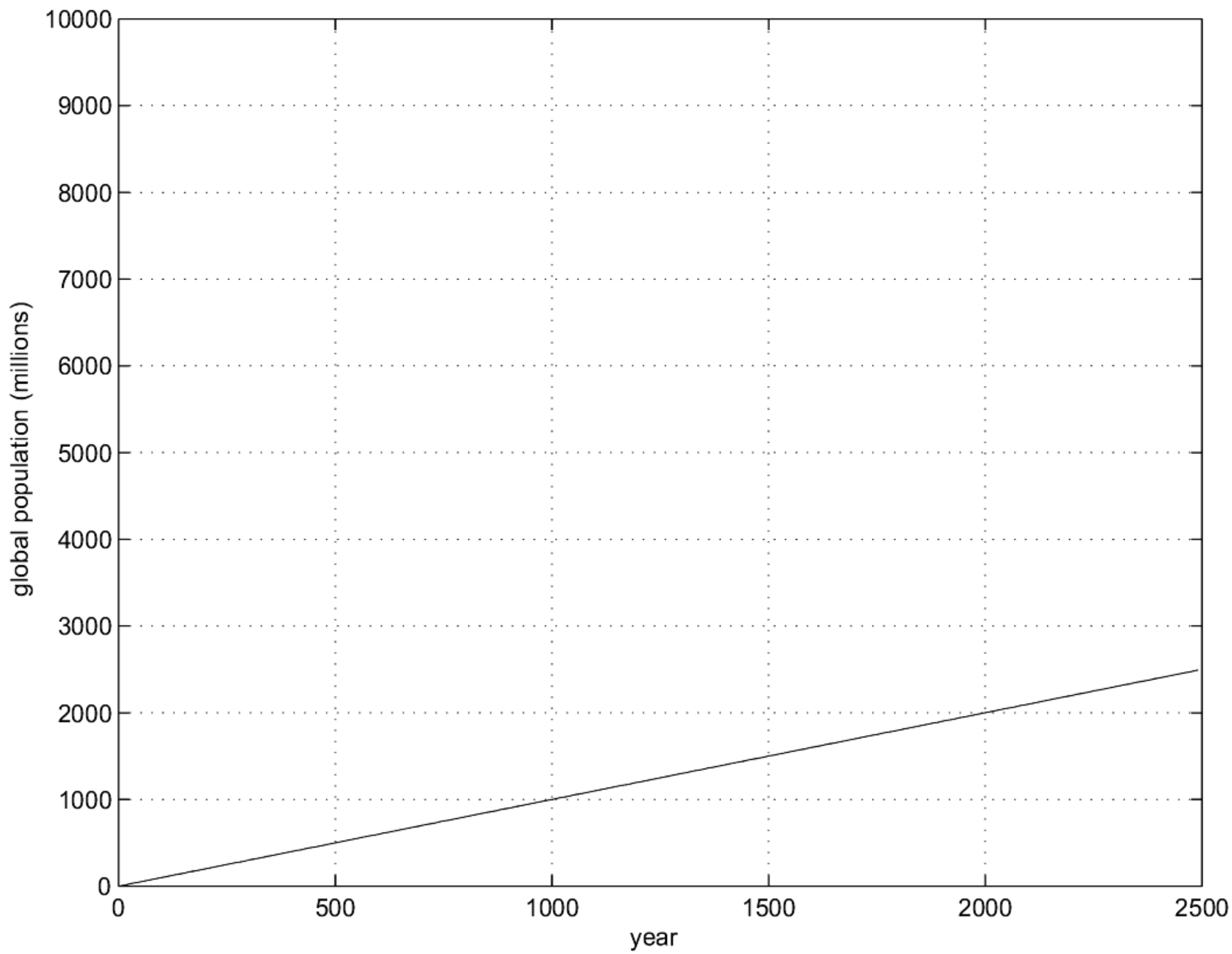
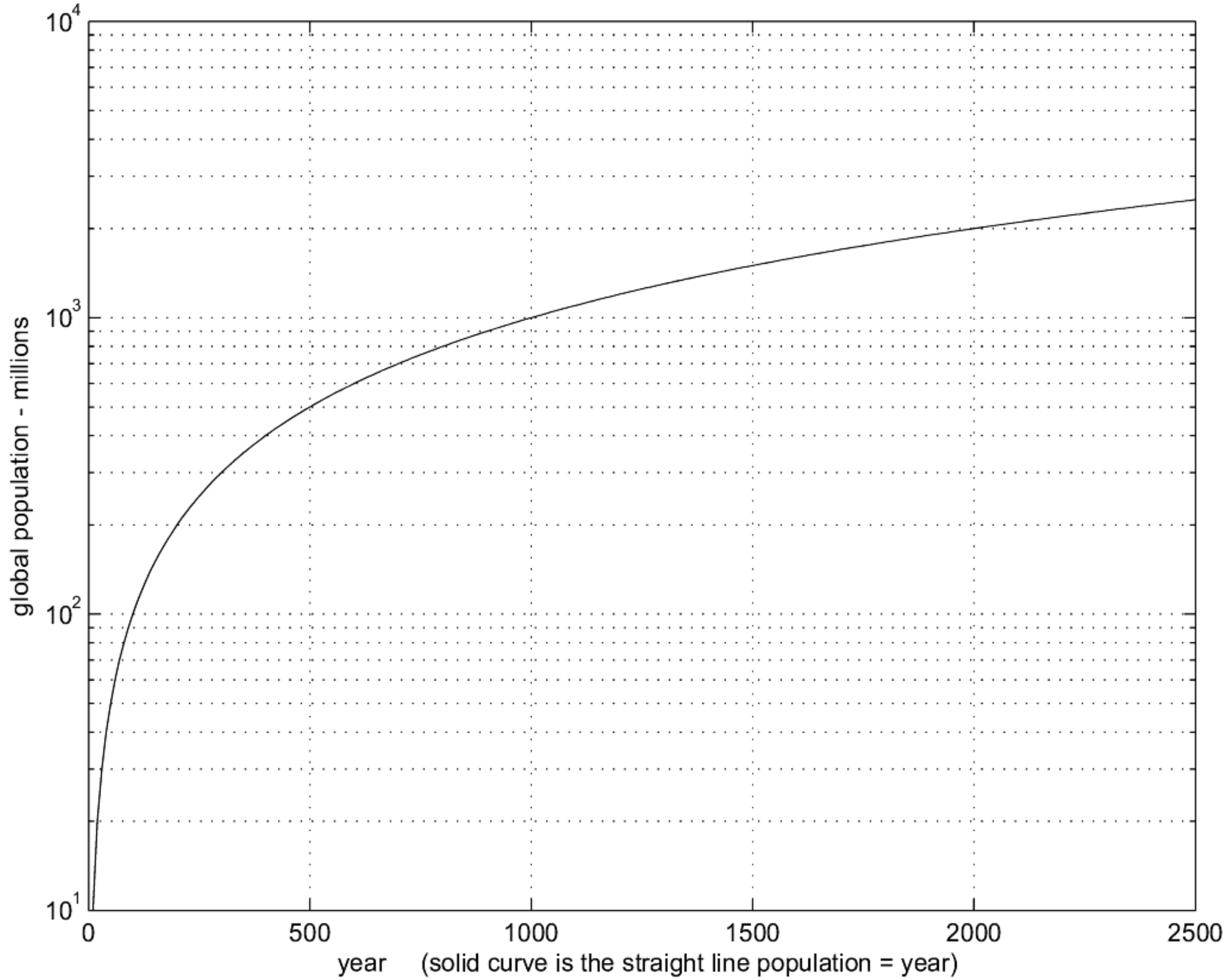
Durand, John D., 1974, "Historical Estimates of World Population: An Evaluation," University of Pennsylvania, Population Center, Analytical and Technical Reports, Number 10, table 2.

Haub, Carl, 1995, "How Many People Have Ever Lived on Earth?"


10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
global population (millions)
0
500
1000
1500
2000
2500
year



10^4
10^3
10^2
10^1
global population - millions
0
500
1000
1500
2000
2500
year (solid curve is the straight line population = year)