Energy Experiments: Background

A document prepared by Peter Rhines (the previous instructor of this class)

Because our textbook relates to energy use, technology and impacts only, we need some more explanation of the science behind the lab experiments. This can help you in doing your final lab-book write ups of the experiments. We come from many backgrounds, so some of you will know most of what follows, others will not. The idea is to move from where you are now (in scientific training) a step or two higher.

Read the sections that are relevant to your experiments with the most attention, and then begin to spend some time reading the others. As you read this everyone should think about the basics of:

- energy conservation (that is using ‘conservation’ in its scientific sense: energy is neither created nor destroyed),
- energy transformation
- energy transmission
- energy efficiency.
- energy ‘evaluation’ (how do we measure it)
- the quantitative idea that ‘work’ transfers energy from one object to another, and ‘work’ (again, by its scientific definition) is equal to force times distance, the force exerted on a body times the distance the body moves. The example below is the heat engine in E4

Notice that some of the energy converting devices in the experiments can be reversed: and electric motor becomes a generator (perhaps generating hydropower); water is split into hydrogen and oxygen gas by passing an electric current through the water, and the reverse reaction is the fuel cell, with hydrogen gas used to make electricity without burning it. It is less easy to see this ‘reversibility’ in the long chain of energy transformation from sunlight to fossil fuel to gasoline engines: in fact we overwhelmingly feel that fossil fuel burning is using up sunlight stored over millions of years. Still some of the chemistry involved can be reversed. Amory Lovins describes chemistry labs as places where fairly natural basic chemical substances are combined to make lots of toxic waste. There is forward-looking university course in a Swiss university where the students reverse this, taking toxic chemicals and turning them back into harmless elemental substances.

As you read through this follow some of the web links, and find more on your own. For example, a simple Google search for ‘diffraction grating’ brings up a wonderful set of descriptions.

E1: SUNS AND RAINBOWS: Examining the solar spectrum.

We think of sunlight as ‘pure’ light or ‘white’ light. Actually it is light radiated from a hot object, and its intensity is a maximum at a particular wavelength…or color…and drops off at other wavelengths. It is the prime source of energy for nearly everything on Earth. There is also some heat coming up from deep in the Earth, some nuclear energy, natural and manmade, and some gravity forces that raise the ocean tides. Even fossil fuels like oil (‘liquid sunshine’) are stored solar energy. The intensity of sunlight is about 1390 watts per square meter, outside the Earth’s atmosphere. The intensity varies a bit over the course of the year because the Earth’s
orbit about the sun is an ellipse, not a circle. This variation ranges from 1345 to 1438 watts per square meter; this variation is one of the driving forces of the huge climate variations that result in the ice ages. Because of the various angles and shadows involved, the average sunlight hitting the Earth over the course of the year is \(\frac{1}{4}\) of the above numbers, averaging 344 watts per square meter of Earth’s surface. That is, if the atmosphere were not in the way. The peak sunshine at noon in Seattle, including the atmosphere’s effects, is roughly 1000 watts per square meter (written watt \(\text{m}^{-2}\)), but this is just for a short time each day.

Have a look at the E2 experiment on prisms and lenses as well as this one. When the sun shines through a prism its path is refracted or bent. The speed of light in glass or plastic varies slightly from the values below, for a variety of wavelengths. This splits sunlight into a rainbow of colors, with correspondingly different wavelengths. We use a low-power red laser to explore the refraction of sunlight one ray, that is one color, at a time.

![Diagram showing solar radiation and absorption]

Fig. 6-1 Spectral radiance \(E_\lambda\) of the sun at mean earth-sun separation. Shaded areas indicate absorption at sea level due to the atmosphere constituents shown (Adapted from Reference 8 with permission).

The figure above shows the actual intensity of light above the atmosphere and a the Earth’s surface, as it varies with wavelength of light\(^1\). Also shown is the curve calculated using theory

\[ M = \frac{2\pi c^2 h}{\lambda^3 (\exp(hc/\lambda kT) - 1)} \]

in units of watts per square meter, per meter of wavelength. \(c\) is the speed of light, \(h\) is Planck’s constant and \(k\) is Boltzmann’s constant, \(T\) is the absolute (Kelvin) temperature. Here \(\exp(x)\) is a way of writing the constant \(e (= 2.718)\) raised to the power \(x\).
by Max Planck for the radiation of a heated body at 5900° C, which is in very good agreement; apparently the sun acts like a simple heated body at that temperature, which is close to what exists at the outer visible layers of the sun. This ideal curve of radiation intensity vs. wavelength is known as black-body radiation.

On the second figures below, the light from various stars differs in the wavelength (and hence color) of its maximum radiation: the cooler stars radiate at longer wavelength, as we have suggested.


Sunlight is not quite ‘pure’; heavy elements in the sun’s atmosphere like iron block certain wavelengths, leaving thin black bands cut out of the spectrum. Also in passing through the atmosphere there are bites taken out of the curve, absorption lines, by interfering molecules everywhere in the air. Water vapor, carbon dioxide (CO₂), methane (CH₄), ozone (O₃) and other gases are excited by light and absorb it; the energy becomes heat or acts to stimulate other chemical reactions, it cannot just vanish! In fact sunlight produces ozone and, in the upper atmospheric region known as the stratosphere this ozone protects us from the ultraviolet parts of sunlight. Ultraviolet rays have wavelengths shorter than violet, are not visible, and are very
damaging to living tissue. Smog (‘photochemical smog’) is something we will meet later in this course, and it is a strong reaction involving oxides of nitrogen, ozone and sunlight. Ironically ozone in the upper atmosphere protects us from ultraviolet rays, yet ozone at ground level, typically intensified along highway corridors, is poisonous to us.

You can see some of these absorption lines with the hand-held spectrometer (do not point it at the Sun) or record them with the spectrograph. Other light sources that you can look at such as incandescent lights, fluorescent lights, and candles have different distributions of energy with respect to wavelength.

The atmosphere removes significant amounts of sunshine, which is partly reflected, mirror-like, back to Space, and partly used to warm the atmosphere itself. The story doesn’t end there because the warmed atmosphere and warmed Earth surface and oceans then radiate at their natural (‘black-body’) temperatures. The temperature of the atmosphere or land or ocean is much lower than that of the sun, so the strongest radiation occurs at much longer wavelength than that of sunlight. In fact it is mostly infrared, longer than red and not visible to the human eye. Some video cameras now are able to see infrared wavelengths and make ‘heat’ photographs. For example an image of a house seen from outdoors in winter shows the ‘hotspots’ like windows and chimney, and is useful in understanding why your heating bill is so high.

The first figure showing the sun’s ‘spectral radiance’ plotted against wavelength of the light shows how the various gases, though present in only tiny amounts, can block the sun’s rays at special wavelengths. Carbon dioxide (CO$_2$) for example is present at about 370 parts per million...that is 0.0037 part of the air is CO$_2$. Note how it is the long wavelengths that are blocked by CO$_2$. This is very important because it also means that the radiation of the ground and ocean back to space, which occurs at long wavelength, will be strongly blocked. Short waves (visible to ultraviolet waves) are also blocked by ozone (see O$_3$ label on figure, near the peak of the sun’s radiation curve).

It is quite a complicated story, but we find in most situations that the short waves penetrate farther than the long waves through water or air. A confusing result however has to be explained: why is the sunset red and the rest of the sky blue? At sunset the light is passing through much more atmosphere than at noon (make a sketch of the rays). So our arguments above would suggest that the long waves should be absorbed. The explanation is that ‘scattering’ of light by air molecules or particles in water is different from the absorption of light. Scattering is more like reflection or ‘bouncing’ while absorption is a total conversion of the light into another form of energy (heat or new chemical bonds). Scattering, it turns out, is more intense for the short waves and lets the longer red wavelengths pass through. An experiment can be done in water, in which a light beam appears blueish from the side yet red when viewed straight on.

To measure the intensity of light (the power or flux of energy, in watts m$^{-2}$) we use a radiometer. It sees all the different wavelengths reasonably well, and gives an electrical output in proportion to intensity of light shining on it. A photographic light meter does pretty well, but its scale is not simply intensity, as it is adjusted for the response of photographic film to different colors. By heating some water and correcting for lost light rays that are reflected off the water surface you can measure intensity quite well.
According to theory, an ideal ‘black body’ radiates energy in proportion to the 4\(^{\text{th}}\) power of its absolute temperature (that is, degrees above absolute zero: Kelvin degrees = Celsius degrees + 273.15). Thus hotter objects are brighter. The area under the different curves showing radiance versus wavelength should be proportional to \(T^4\). As you know from sitting by a fire, the color of radiated light varies with temperature. The flame itself may be blue near the base where it is hottest, then redder where it is cooler, finally becoming invisible (infrared) where it is cooler still. The wavelength of the brightest radiation (the peak of the radiance curves in the figure) is \(\lambda_m\), given by

\[
\lambda_m = \frac{2898}{T} \ \mu\text{m}
\]

The units \(\mu\text{m}\) are microns (millionths of a meter), and \(T\) is in absolute degrees (K). The ‘color’ or wavelength of the radiation varies inversely with the temperature of the radiating body.

Different stars in the night sky have different colors, because of their different temperatures (the second figure above). The infrared thermometer that is being used for experiments \(\text{E6 and E7}\) uses this characteristic to sense the temperature of items that you point it at.

As mentioned above, when sunshine is absorbed by a planet like Earth, it then radiates back to Space with a spectrum that corresponds to its own temperature (much lower than the sun’s). According to the formula just above, this radiation will have a much longer wavelength, probably infrared, or invisible to the eye. We can see the planets not because of their radiated infrared light but because of directly reflected sunlight (the ‘mirror’ effect).

These are key ideas for understanding our atmosphere, and global warming.

* Planck’s formula describing the ideal solar radiation above the atmosphere is

\[
M = \frac{2\pi e^\frac{hc}{\lambda^3} \exp\left(\frac{hc}{\lambda kT}\right) - 1}{\lambda^3}
\]

in units of watts per square meter, per meter of wavelength. \(c\) is the speed of light, \(h\) is Planck’s constant and \(k\) is Boltzmann’s constant, \(T\) is the absolute (Kelvin) temperature. here \(\exp(x)\) is a way of writing the constant \(e (= 2.718)\) raised to the power \(x\).

**E2. LENSES AND MIRRORS:** Concentrating energy and taking apart sunlight, ray by ray

*Basics of light and lenses.* Light is a wave and yet it is known also to act like a stream of particles – photons – which are individual packets of energy. For present purposes think of light as a wave. The light’s wavelength is the distance between adjacent ‘wave-crests’, or maxima of the sine-wave describing the light’s amplitude, and it ranges from 420 to 700 nanometers (1000 nanometers = 1 micrometer or 1 micron). This seems a very short wavelength, yet it is not really too far out of our common experience. The wavelength is much larger, for example, than the size of individual atoms (very roughly 0.1 nanometer, or 1 ‘Angstrom’).

Art comes in at this point: the different wavelengths of light correspond to distinct colors. Shorter wavelengths (say 450 nanometers) are blue light, longer wavelengths (say 650 nanometers) are red. The *spectroscope* used in experiment \(\text{E1}\) breaks down white light into its component colors, each of which corresponds to a particular wavelength of light. The light first passes through a slit to turn it into a narrow beam, and then is separated into its different colors.
using a *prism*, a piece of glass or clear plastic with triangular cross-section. The result is a that a rainbow is created out of white light.

A key property of waves of all kinds is the relation between wavelength and *frequency*. Frequency, call it $\sigma$, in cycles per second, is just the number of waves per second seen passing a fixed observation point. A pendulum swings back and forth periodically, rhythmically repeating. The repetition time, the *period* is just $1/\sigma$. A light wave is wavy in both space and time and the frequency and wavelength of the waves are related by

$$ c = \sigma \lambda $$

where $c$ is defined as the *wave-speed*. It is just the speed of the moving wave-crests. For light $c = 3 \times 10^8$ meters per second in free space (a vacuum). Light is slowed as it goes through glass or water or plastic. This formula is really just ‘distance = speed times time’ written for a wave (for time equal to the wave period, the distance traveled is one wavelength, from one wave crest to the next).

*Refraction.* We described above how the speed of light, $c$, has its maximum ($3 \times 10^8$ meters per second) in empty space or a vacuum. When it passes through glass or plastic or water its speed is slower (as an electromagnetic wave it excites the atoms of the material it is passing through, slowing it down). The *index of refraction* of the material, call it $n$, is

$$ n = c_0/c $$

where $c$ is the speed of light in the material and $c_0$ is the speed of light in a vacuum, as in outer space. Some values for the index of refraction:

<table>
<thead>
<tr>
<th>Material</th>
<th>Index of Refraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacuum</td>
<td>1.000000</td>
</tr>
<tr>
<td>air</td>
<td>1.000277</td>
</tr>
<tr>
<td>ice</td>
<td>1.31</td>
</tr>
<tr>
<td>water</td>
<td>1.333</td>
</tr>
<tr>
<td>ethyl alcohol</td>
<td>1.362</td>
</tr>
<tr>
<td>glycerine</td>
<td>1.473</td>
</tr>
<tr>
<td>polystyrene</td>
<td>1.59</td>
</tr>
<tr>
<td>crown glass</td>
<td>1.50-1.62</td>
</tr>
<tr>
<td>flint glass</td>
<td>1.57-1.75</td>
</tr>
<tr>
<td>diamond</td>
<td>2.417</td>
</tr>
</tbody>
</table>

which will give you an idea why diamonds are so pretty to look at.

Now, as a ray of light in air hits a piece of glass, the wave-crests move from air into the glass. When a wave-crest has one end in the glass and the other in air, the part in air is traveling faster, so it catches up with the slower part in the glass region. You can see that this will rotate the wave-crest and the light ray which is always perpendicular to it. The same thing happens with ocean waves at the beach: why else would most all of the waves hitting the beach line up nearly parallel with it? An airplane ride will convince you that as they slow down in shallow water, the wave-crests turn toward shore. The rule describing this turning of the wave-crests is

$$ \sin \theta_1 = \frac{n_2 \sin \theta_2}{n_1} $$
Experimenting with one light ray at a time (using the laser which has just one wavelength (i.e., one color of light), you can see this change in path of the light ray moving from air to glass or air to water. We sometimes put the glass prism in water simply so that you can see the light beam (due to particles in the water); the same can be done in air by sprinkling chalk dust in the beam.

*Lens.* When a light source, say a simple light-bulb, sends beams in all directions some of them can be captured by a piece of glass curved to form a lens. Where the surface of the lens is tilted most (the outer, thin part) light rays are bent most; where the surface of the lens is tilted least (the middle) light rays are bent least.

The result can be a convergence of rays after they pass through the lens. The rays can all come together at a point; when the light bulb is a long distance away, this point is at the focal plane of the lens. It’s pretty easy to see how this works for a small light-bulb, a ‘point’ source of light. Notice that on a piece of paper held at the focal plane, you will see a dot of light. Hold it anywhere else and you will see a ‘spot’, perhaps a very big one. The lens concentrates the light at one place. If you put a kernel of popcorn there you may be able to pop it (well, if the light source is the sun, or as bright).
As you move the light source closer to the lens, so that its rays are less and less parallel, they pass through the lens and focus farther away. Finally when the light source is at the focal plane, the rays coming out the other side are parallel...they focus only at 'infinity'. This says that optics are 'reversible'; you can run the light rays in the figure above either forward or backward. The general rule is that the distance of the light source from the lens, \( d_1 \) and distance of its image on the other side, \( d_2 \), are related by

\[
\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}
\]

which holds for fairly thin lenses. Here \( f \) is called the focal plane distance or focal length. Notice that as one of the distances becomes very large, the other becomes equal to \( f \), which is what we have seen just above.

**Images.** An important application is in creating images for using such devices as cameras, slide projectors and eyeglasses. An image is a ‘copy’ of a 3-dimensional object onto two dimensions: a picture. The point of light from a single distant source is a simple image, seen on a piece of paper at the focus of the lens. If the light bulb used above has some interesting shape, and is not just a point of light, that shape will reappear at the focal plane. If you had just two beams of light from two far-away light bulbs, they would be incoming at different angles, like two stars in the sky. You could trace the rays, parallel from each of the stars, through the lens, and see them focus at the focal plane. These two points of light are a simple picture...an image. Add a few more and you begin to have a ‘picture’. If your retina, the sensitive surface of the bundle of nerves at the back of your eye, lies along the focal plane of your eye’s lens then our brain can make sense of the image (even though it’s upsidedown!). The points of light are focused on the retina and send a few discrete messages that your brain can interpret as the image. To ‘see’ close-up objects in focus you have to bend the incoming rays more, according to the above formula. As you get older your eye’s lenses become less flexible, their focal length tends to be less controllable by your muscles and the world gets blurry. Even if you once had perfect vision, extra lenses become necessary.

Look through a magnifying glass and think about what you see, with objects at various distances away...note the tendency for images to be upside-down at some distance, and right-side-up at others. In addition to making eyes and cameras work, lenses make projectors possible. There, the film image sits between the light source (arriving roughly as parallel light) and a fat lens. A large image, much larger than the original on the film, will be in focus at a specific distance away (use the two-light-bulb case above, to think about this). Where is the film image relative to the focal plane of the lens when the projected image is quite far away?
Take a light bulb, and hold a lens (magnifying glass) between it and a sheet of paper. As you move the lens from near the bulb to near the paper you see a sharply focused image of the bulb at just two places...two distances from the bulb. The sizes of the two appearances of an image are different. This is another example of the 'reciprocal' principle above. One solution of the equation above for the distances $d_1$ from light source to lens and $d_2$ from lens to focal plane leads to another solution, just reversing the roles of $d_1$ and $d_2$.

**Mirrors.** A mirror reflects light rays, the incoming and outgoing rays being symmetric about a line perpendicular to the mirror’s face. If the mirror is bent into the shape of a parabola, a set of parallel rays (coming from a distant point of light) will all pass through the focal point (focus) of the parabola. A ‘shaving mirror’ magnifies your reflected image. Think about the rays of light forming this image (using a light bulb with a bendable piece of mirror try to focus the reflected light to as small as possible an image).

If two pieces of mirror are put together and held at a small angle, to form a ‘corner’ then a light ray (laser beam) bounces back and forth between the two mirrors. If the mirrors are held at a small angle, a very bright spot forms in the corner. Trace some light rays, each bouncing off the mirror according to the symmetric reflection law above and look for regions of concentration.

Thus both lenses and mirrors can concentrate light, gathering all the rays that hit its surface and sending them through the focal point. This is a form of focusing of energy that can be put to use in designing a collector of solar energy...as in a solar cooker or solar ‘furnace’ or solar driven photovoltaic electricity plant.

**E3 A MODEL RIVER: Energy conversion in a propeller-driven water channel**

A very common problem in the energy world is the generation of electricity. Why not generate heat and transmit it to where it is needed? It happens that transmitting electricity, as alternating currents with very high voltage, can be done with relatively little energy loss (compared with sending a pipeline of hot water across the country which would not be efficient). Electricity is one of the main elements in our energy profile. We have a ‘race-track’ shaped water channel in which are mounted some small propellers. These propellers are driven by small electric motors, connected to a power supply that provides a controllable voltage and current. If this were a river we could let the moving water spin the propellers, which would turn the coils of wire in the motors; in turn electricity would be generated as the coils of wire passed the magnets fixed near them: *an electric motor can be turned into an electricity generator, running the energy conversion backward.* This describes hydropower, which is a major and economical source of electricity in the northwest.

Running the motors normally with electricity we drive the water round the channel. To investigate the conversion of electrical to mechanical (kinetic-) energy of the flowing water, we need to find quantitatively how much electric power is being used. The formula is

\[ \text{power (P, in watts)} = \text{electrical current (I in amperes)} \times \text{voltage (V, in volts)} \]

or

\[ P = I \times V. \]
We are using direct current (d.c.) produced by a d.c. power supply connected to the propellers. With direct current the voltage and current are steady, representing a flow of electron charge along the wires (alternating current, a.c., which we commonly use for power, has its voltage rapidly varying between +115 volts and –115 volts, 60 times per second). One advantage of a.c. is that it is easily converted from one voltage to another, using a transformer. High voltage lines carry it across country and it is stepped down to 115 volts for your use. With a.c. some of the energy is in an electromagnetic field surrounding the wire, and less in the electron flow in the wire.

Kinetic energy (KE) appears in the water, equal to

\[ KE = \frac{1}{2} m U^2 \]

where \( m \) is the mass of the water and \( U \) is its speed. When the propellers start up they accelerate the water until a more-or-less steady flow is reached. It is during the start-up period, as the water is accelerating, that we can do a simple calculation. How much of the electrical energy appears as mechanical energy? Where has the rest gone?

A clue to the lost energy is found in the steady state that is reached after a few minutes. We are still putting electrical energy to work, spinning the propellers. They are working hard. Yet the water is no longer accelerating on average. So in this steady state the energy put in is being totally lost….to friction which heats up the water and the sides of the channel. We are slowly warming up the water and the lab. It is very subtle because heat energy is very ‘rich’ or concentrated compared with mechanical energy in our practical world. It is difficult to warm up you cold cup of coffee by stirring but you could do it. This contrast of the ‘richness’ of various forms of energy is at the heart of our schemes for using energy in society. We may store it as ‘liquid sunshine’ (as oil has been called), for hundreds of millions of years. But it came from the sun.

It is interesting to think about hydropower, using some of these basic ideas. The mechanical energy of the flowing water can involve both kinetic energy (due to motion) and potential energy (due to height of the water above some reference height). Potential energy, expressed as the product \( mgh \), is given up to supply kinetic energy (\( h \) is the height difference experienced by the moving water, falling downhill, and \( m \) is its mass and \( g \) the constant acceleration due to gravity, 9.8 meters per sec\(^2\) which would be experienced by Galileo’s ideal rock falling from the Leaning Tower of Pisa).

In a flowing river, as in a swinging pendulum, these two forms of energy trade off with one another. Yet, there is an important difference between pendula (pendulums) and rivers: the pendulum swings back and forth for a long time, rhythmically trading kinetic and potential energy back and forth, rising and falling, before coming to rest. In a river, the water flows downhill and constantly, rapidly, converts energy one-way from potential to kinetic. Then it loses the mechanical energy at the river bed to heat. Actually throughout the water the flow is usually ‘turbulent’…chaotically moving in whirling eddies…and this chews up kinetic energy, turning it to heat. Is this what we see in the lab water channel?

Taking energy out of rivers, as they flow downhill, thus may involve a lot of energy loss (slightly warming up the water and riverbed). Looking into the efficiency of modern
hydropower plants, and the way rivers are dammed to make them work, can profit from these ideas.

**E4 A HEAT ENGINE: An engine using heated air to make mechanical energy**

Energy conversion from one form to another is the essence of an ‘engine’, whether in a car or electric device. Many such engines turn heat into mechanical energy (which a car does once the chemical energy of gasoline has been converted to heat). In environmental studies we are sensitive to the damage that so many millions of gasoline engines are doing to the atmosphere, and we can see in our lab a promise for a much cleaner future in the hydrogen fuel cell of experiment E8. It is worth understanding some of the principles of a basic small engine run by heat, because they will also apply to a much bigger heat engine: the atmosphere/ocean circulation. We will see, again and again, the important idea of ‘efficiency’, waste energy, high-quality and low-quality energy.

Here goes: if you squeeze a gas it becomes warmer instantly. This happens because ‘squeezing’ means exerting a force and changing the volume of gas. Compressing the gas requires work, which transfers energy from our muscles to the gas; the work goes into internal heat energy of the gas and so it warms up. Thinking of a gas like air as billiard-ball like molecules flying around, bouncing off each other and bouncing off the walls of their container, you can imagine that squeezing the gas could make the molecules move faster. A baseball hitter swings and the ball (the molecule) acquires more speed, and in a different direction, than it had before.

We can run this argument the other way. If you heat a gas (say with a candle), it will expand if it can, or if it can’t its pressure will increase. Now ‘pressure’ is related to the speed of the molecules (actually the square of the speed), and so is the kinetic energy of the molecules, and so also is the temperature. If the gas is allowed to expand it could lift a weight at the same time. Lifting the weight means giving it some extra potential energy. Or, if the gas expands it could do some useful mechanical work as in turning a crank or a propellor.

All of this agrees with the conservation of total energy:
change in thermal energy of gas =  heat added - work done on surroundings
(see also the discussion for the candle experiment, E5).

A heat engine takes these two ideas and combines them. Here is an example:
1. heat an enclosed volume of gas at constant pressure (with a weight sitting on top of it…see sketch): the gas expands, lifting the weight
2. stop heating and remove the weight, which is higher than when it started; the gas expands some more, with no heat energy input (‘adiabatically’).
3. now cool the gas down, removing just the same amount of heat added in step 1. It will shrink in volume.
4. place another weight on top, which will compress the gas further.
We are now back where we started, after a 4-stage cycle.
During each cycle we lift a weight up…creating mechanical energy from thermal energy. The key is to expand the gas at high pressure and compress it at low pressure. The figure below is a wonderful way to represent this heat engine cycle. Plot the pressure of the gas, $P$, on the vertical axis and the volume of the gas, $V$, on the horizontal axis. This so-called $P$-$V$ diagram will show you a closed curve as the engine moves cyclically. The heat-engine curve can differ in detail, but it always has one key feature: gas expands when pressure is relatively high and contracts when the pressure is relatively low. When the weight is sitting on the gas, pressure is higher, when the weight is removed, pressure is lower.

How much mechanical energy is this little engine producing? That is how much work can it do? As we described earlier, lifting a weight to a height $h$ requires an amount of work; work is quantitatively given by force times the distance traveled by the forced object, and so

the power produced by the engine is the rate of doing work, or work during one cycle divided by time to carry out one cycle.

This work creates potential energy which can then be utilized (by dropping the weight for example) to make kinetic energy.

In the gas experiment, the force per unit area is the pressure $P$, and in the two-dimensional sketch above, the distance is proportional to volume change, call it $\Delta V$. Since force times distance is proportional to $P$ times $\Delta V$, during each piece of the cycle the work done is proportional to the area beneath the $PV$ curve. When it travels left the sign is negative, subtracting off from the work done on the rightward part of the cycle. Thus

work is also given by the pressure times the change in volume of the gas
and

the total work done during one cycle of the heat engine is equal to the area inside the closed curve on the $P$-$V$ diagram.
By the way check the ‘units’ of P times V: they are the same as the units of force time distance, which is reassuring. A surprising feature of this heat engine is that it doesn’t really ‘use up’ any heat energy. That energy goes from a warm place (like the candle) to a cold place (whatever is cooling the gas down, say an ice cube). It goes to a lower temperature (see diagram below). We sense that it is the contrast in temperature, cold next to hot, that makes engines run. Having a lot of hot things around, all the same temperature, would not allow us to do much of anything. Plenty of energy but since heat flows from a hotter body a colder body and it is heat flow that makes things go, temperature differences also make things go. Each time we run our heat engine we bring the Universe close to a uniform temperature. When and if we get there, there will be no more heat engines.

The statement above in italics is the simplest form I know of the ‘2d Law of Thermodynamics’ that talks about a quantity called entropy. It has deep philosophical connections as you might guess from the paragraph above. It also has a connection with patterns, information, chaos…lots of ideas about ‘order’ and ‘disorder’. You may encounter these ideas far away from science, and find them interesting. The Universe, we imagine is heading from order to disorder, from hot and cold bodies to a single, cool, uniform temperature. We as human beings represent very unlikely combinations of atoms; we are examples of organization, islands of ‘low entropy’ trying to fight off the forces that would disorder us, turning us into seas of randomly distributed atoms. It is enough to build a religion on.

The Stirling Cycle engine in the lab takes this kind of heat-work-cycle and puts it all together. It is complicated to figure out all its parts, but worth the effort. Notice that ‘mechanical energy produced’ or ‘work done’ by this engine is pretty hard to see: it is in the mechanical energy of the rolling marbles in the glass tube back and forth (which you can measure). We could instead drive a propeller which can be observed and measured…yet each cycle the mechanical energy is lost as the marbles hit the end of the tube and stop (ironically losing their kinetic energy to a slight amount of heat). That’s a confusing ‘error’ in this demonstration. But the rest is convincing. The Stirling engine has the property that, without the candle driving it, it is nearly balanced like a see-saw, between tipping one way and tipping the other. It is nearly an ‘oscillator’, and when the candle heat is added that is just enough to make drive the oscillation.

It happens that this PV cycle can be quite efficient in a practical application. Let’s define efficiency as the mechanical work done, divided by the heat energy input to the engine or ‘work out’/heat in, both in units of power (watts). For the record, the heat engine has a maximum possible efficiency (the ‘Carnot’ efficiency) of

\[
\text{efficiency} = \frac{T_1 - T_2}{T_1}
\]

=⇒ heat in

=⇒ work out

=⇒ heat out

where \(T_1\) is the temperature of the heat put into the engine (the ‘intake’ temperature) and \(T_2\) is the temperature at which it is taken out (the ‘exhaust’ temperature). What this represents is the
mechanical power (energy per unit time) produced by the engine, divided by the heat energy flowing into it. To find the heat energy going in, look at the discussion of the ‘candle’ experiment, E5 where we show that heating a gas increases its thermal energy by an amount equal to the ‘specific heat capacity’ (roughly a constant) multiplied by the temperature change. On the PV diagram, if the gas is an ‘ideal’ gas like air, without moisture in it, then P and V are related by the equation of state,

$$PV = nRT$$

where T is temperature and n = mass of gas divided by molecular weight. Thus curves PV = constant on the diagram correspond to constant temperature curves. By making intake and exhaust temperatures very different, we make the heat engine’s cyclic curve larger (reaching larger range of temperature) and thus it will enclose more area, and correspond to more efficiency.

This efficiency is greatest if the difference between intake and exhaust temperatures is large. It shows why we want to concentrate sunlight before using it to drive an engine: we will get more mechanical energy out. Your car is more powerful on a cold winter’s day than a hot summer’s day (other things being equal).

The heat engine takes heat from hot regions and moves it to colder regions, taking out some mechanical energy on the way. Like many of our labs, the heat engine is ‘reversible’, meaning that it can run backward. We can put in mechanical work and make heat flow away from a cold region to a hot region. This is a simple refrigerator!

**E5 MY CANDLE BURNS AT BOTH END: Measuring the useful energy content of fuels (energy conversion, hydrocarbon to heat)**

In the getting started document we noted that sunshine creates plant life, and there is an energy flow that accompanies the chemistry of photosynthesis. Once the energy is stored in chemical bonds within the plants, some of it remains even as the plant decays. Solar energy becomes entombed as fossil fuel: oil, coal, and gas. The energy released by burning a few common fuels is

- hydrogen 142 kJ/g
- methane 55
- octane 48
- methanol 23
- paraffin C_{20}H_{42} 42

Here the units of energy content are kilojoules (thousands of joules), KJ, per gram of fuel (or what is the same number, megajoules per kilogram of fuel). Burning a candle produces heat (candlewax should have about the same energy as paraffin). As it burns, the candle loses weight. By heating some water at the same time we can estimate how much energy is converted by burning the candle. Heated water is a nice ‘energy standard’ that you should become familiar with. Experimentally it is found that

Supplying 4.185 Joules of heat energy to 1 gram of water will raise its temperature by 10°C (1 degree Celcius), if the water is near 15°C. This number 4.185 J/g °C (Joules per gram per degree C) is known as the **specific heat capacity of water (at constant pressure)** and is
represented by the symbol $c_p$. We can use the water-heating experiment to measure the energy in sunlight or candle wax or oil (and we should even be able to do that frustrating experiment of violently stirring our coffee, and measuring the warm-up).

We should put this energy transformation in the context of conservation of energy…the 1st law of thermodynamics. It’s simplest for the case of a gas rather than a liquid like water. This conservation law states that

\[ \text{Change in internal thermal energy of a gas} = \text{heat flow into or out of the gas} - \text{work done by the gas on its surroundings (by expanding or contracting)}. \]

See also the notes for the heat-engine experiment E4. Gases like oxygen, hydrogen or mixtures of gases like air (air is 78% nitrogen and 21% oxygen, plus traces of other gases) also have a simple expression for their internal, thermal energy: to a good approximation, this energy is simply proportional to temperature

\[ \text{internal energy} = C_v T \]

where now $C_v$ is called the specific heat at constant volume. Numerically it is given by

\[ C_v = 0.717 \text{ J/g}^0\text{C} \]

so that water holds substantially more heat than an equivalent mass of air (comparing 0.717 with 4.185 above). This relationship tells how much the gas will warm up when heated, if the gas is confined so that it keeps the same volume while being heated. If it is allowed to expand, the expansion does some work (the amount being $P \Delta V$ where $P$ is pressure and $\Delta V$ is the volume change of the gas; see the heat engine, E4) and then the internal energy change is less than the heating that goes into it (so as to obey conservation of total energy):

\[ \text{heating of gas at constant pressure} = C_p T \]

where $C_p \equiv 1.4 C_v$.

So now our conservation law above becomes

\[ C_v \Delta T = \text{heat added} - P \Delta V \]

An interesting exercise is to imagine heating your house by storing the summer’s heat, and releasing it in winter. Suppose you make a ‘swimming pool’ and use a solar hot-water heater to heat the pool all summer long. Keep it well insulated. Think about how big a pool should be to give you enough heat to get through the winter.

Notice in the list above how hydrogen is the star performer in the energy race. When it is burnt (oxidized…combined with oxygen) it gives off much more heat (per unit mass) than any other fuel. Its chemical reaction combines hydrogen gas and oxygen gas to make energy and water. No pollutants! In a fuel cell where hydrogen gas generates electricity directly, the energy yield is a bit lower but not much.

**E6. DOES HEAT FLOW UPHILL: Conduction and convection in solids and fluids.**

When heat energy is transferred to a fluid, the fluid reacts differently in terms of absorbing and spreading that heat within itself, depending on which direction the heat is coming from. There is a very good reason to heat a fluid from the bottom other than the fact that it’s easiest to
contain it in a vessel (pot, cup, beaker, etc.) and put that vessel on a heat source. Fluids (even gases) naturally move heat throughout themselves in a process called convection if heated from below, but not if heated from above. Another form of heat transfer is conduction, a very much slower process, which is direct transfer from one molecule to the next. This occurs between parts of the system that are physically in contact as from the stove top to the metal pot. Radiation is the third form of movement of heat: solar energy is transmitted through space to Earth in the form of light electromagnetic radiation. Convection and conduction actually interact in interesting ways; both are present in a heated pot of coffee.

The law of heat conduction states that heat is transported at a rate

\[ k \frac{\Delta T}{\Delta X} \]

where \( k \) is a constant property of the material, known as the conductivity, and \( \Delta T \) is the temperature difference over a distance \( \Delta X \). In the experiment you look at the conduction of heat in an aluminum bar, and in water heated from above and below. Despite the fact that the conductivity, \( k \) is much greater than aluminum than for water, the water may in some circumstances move thermal energy much faster than it conducts through aluminum. The key of the experiment is to see what those circumstances are, and what the fluid does to move heat quickly by ‘convection’ or just by flowing.

All of our uses of energy involve conversion from one form to another, transport or ‘transmission’ from one place to another, storage until we need it, and conversion again (perhaps to push your car along). Here we want to explore transport or transmission of heat energy in a fluid. The results apply to many flowing systems, as we shall see later in Water and Air unit of the course. We want to see energy transport working in both natural (out-door) and man-made energy cycles.

Convective energy flow is all around us. The hot-water in our faucets comes from a convectively heated tank (gas or electric input) and the cumulus clouds in the sky are often from air convecting up from the Earth’s surface. Radiation is felt as sunshine, even when it is cloudy. Conduction is clear when something ‘feels’ hot to the touch.

Conduction through a solid is more of a molecular process where energy is absorbed and transferred by vibration of the molecules within the solid. You can heat a piece of metal at one end and find that heat is conducted to the other (try this). The speed and efficiency depend on a number of factors, the most important of which is the temperature difference between the metal and what’s heating it, but also the kind of metal.

One of the big issues in human use of energy is in transforming it from one form to another efficiently. Most of the energy used by people starts out as sunlight. As you can see from the figure under experiments E1 and E2, this is absorbed by water quite effectively at a number of wavelengths, especially in the infrared (redder than you can see, wavelengths of 0.7 micron or greater). This is true for water in the atmosphere as well, which is why water vapor is important to regulating the temperature of the atmosphere. All this suggests that you could heat water with sunshine from above, but coming from the top may reduce it’s effectiveness: this should come out in the experiment. Think about what you discover here in terms of design of a solar box cooker to heat water or food as effectively as possible. It’s a little different from putting a pot on
a fire. You can compare two test-cases: heating a fluid from below and heating it from above, and begin to assess the differences in energy transport, and why they occur.

**E7 A SOLAR POND: Energy conversion and storage, solar to thermal**

Sunshine falling on the Earth heats it: by conduction, heat is carried down into the soil. But this conduction is very slow, and the distance the heat travels is only a meter or so in a year. If you live in a cold climate, you bury your water pipes a meter and a half or so deep so they won’t freeze in winter. Permafrost in the Arctic is a frozen layer that never thaws out, even in the warm Arctic summer, because that layer ‘averages’ the air temperature throughout the year, and summer’s heat can’t reach it.

Let us imagine a way to trap some of the sun’s heat so that it can be used to do useful work. Water is a useful medium, because you can move it, and its heat, readily from place to place. But the sun shining on a lake warms a thin layer at the surface and at the same time much of that heat is given back to the air above, in the form of evaporation and clouds. In experiment E6 we learn that convection is an efficient means to transport heat energy upwards - hot fluids are less dense and rise. Thus, even if we could trap heat at the bottom of a ordinary pond (perhaps by having a black bottom that absorbs – converts to heat - the suns radiation that makes it to the bottom), convection would transport it back to the surface.

However, we can stop convection if the bottom of the pond is also more salty than the overlying waters. The two plots below show how the density of pure water varies with temperature and how the density of salt solutions varies with salinity. The salinity in percent is the weight percentage of the water-salt solution that is made up of salt. Thus if you dissolve 3 g of salt (sodium chloride NaCl) in 97 g or 97 cm$^3$ of water you have a salinity of 3%. You can see from comparing the plots that adding a just few weight percent of salt has a bigger effect on density than increasing the temperature from 0 to 100ºC. A 10 weight percent salt solution at 99ºC will be denser than fresh water at 20ºC.

![Graphs showing density of water and salt solutions](image)

People in the Middle East, Asia and even Florida build shallow ‘solar ponds’ in which a layer of very salty water lies at the bottom. When the sun warms this pond, the heat is trapped and can be used for purposes such as heating for recreational and industrial applications. In the lab,
trying this out will give an idea about radiative heating, the distance that sunlight penetrates into a fluid, and other forms of heat transfer. You may not be able to reach the very hot temperatures of the real solar ponds (as hot as 99\(^0\) C) but the trapping of deep heat should be visible.

An example from India:
http://www.teriin.org/case/bhuj.htm

E8 YOUR NEXT CAR? THE HYDROGEN FUEL CELL: Energy conversion, chemical to electrical and reverse.

‘De-carbonizing’ our energy sources is an important trend that has been going on for much of the last century. This means changing from carbon-rich fuels like fire-wood to coal then even better to oil and natural gas, which in each case meant more energy with less carbon…both in the form of carbon dioxide gas and incompletely burned fuel…carbon monoxide and soot.

The fuel cell offers tremendous hope for clean fuel. Here pure hydrogen is burned with oxygen to make energy. The only pollution byproduct is pure, fresh water. One problem is, where to get the hydrogen? It must be generated somewhere, and this may involve a fossil-fuel burning generator. But the hope is that a centralized fossil fuel plant can have some or most of its carbon ‘scrubbed’ from its smoky exhaust, and will be cleaner than a decentralized burning of fossil fuel in every car and truck and lawnmower. Also, further in the future, clean renewable energies like solar, wind, hydropower, could generate the hydrogen gas. Thus the fuel cell technology is a way of improving the efficient conversion and transport of energy from source to final energy product, the kinetic energy of your car.

Electrochemistry is a complex part of the science, but we can see its results in a simple experiment. It is so important for future power sources, that is engines, that we must include it. The following discussion may not be useful to those who have not studied any chemistry, but we include it for those who have.

Different chemical species have different affinities for electrons. This difference can drive electrical currents, which are a very powerful form of energy. The exchange of an electron, call it \(e^-\), between two chemical species dissolved in a fluid is a called a redox reaction: reduction-oxidation. Reduction of a species adds an \(e^-\) (making the charge less positive) while oxidation takes away an \(e^-\) (making the charge more positive).

An example is metallic copper combining with silver ions to make copper ions plus metallic solid silver:

\[ \text{Cu (solid)} + 2 \text{Ag}^+ \Rightarrow \text{Cu}^{2+} + 2 \text{Ag (solid)} \]

This happens spontaneously because silver (Ag) ‘wants’ electrons more than copper (Cu) does, and rips them away from the copper atoms. This affinity is measured as an electric potential in volts. So this is a ‘battery’ in which electricity flows and the copper electrode is eaten away.
Anode oxidation  
Cu(solid) => Cu^{2+} + 2e^-  

Cathode reduction  
Ag^+ + e^- = Ag (solid)

Electrolysis. We can force a voltage \( V \) on the system rather than let the chemical system determine the voltage. This will forcibly move electrons. If the voltage is high enough, we get a non-spontaneous, forced reaction:

\[
2\text{H}_2\text{O} \text{ (liquid)} \Rightarrow 2\text{H}_2 \text{ (gas)} + \text{O}_2 \text{ (gas)}
\]

This suggests that you should get twice as much hydrogen gas as oxygen gas. In the lab experiment you collect the gas and can demonstrate that hydrogen burns.

The electric current is aided if the water is made more conductive (pure fresh water does not conduct electricity very well). By adding some sodium hydroxide or sodium carbonate we then have ions (charged pieces of the sodium hydroxide molecule which freely splits or dissociate in the water). Now with a good current passing through the water, water molecules are split into ions \( \text{H}^+ \) and \( \text{O}^- \), which then are attracted to the electrode of opposite charge (hydrogen ions cluster on the negative electrode, known as the ‘cathode’ while oxygen ions cluster on the positive electrode, the ‘anode’). There they recombine into gas molecules: 2 hydrogen atoms making a molecule of hydrogen gas and 2 oxygen atoms making oxygen gas.

Transferring some hydrogen to another set of electrodes shows that the reaction can go in reverse: this is the hydrogen fuel cell. The electrodes we are using are made of titanium. The metal is very important: it is a ‘catalyst’ for the reaction (making it possible, yet not appearing in the chemical reaction above). A voltmeter attached to the electrodes shows a definite voltage change when the hydrogen gas comes in contact with the electrode. This is a small but accurate ‘fuel cell’ producing electricity from hydrogen.

The demonstration fuel cell that drives a small propeller operates on a few drops of 3% methyl alcohol (methanol) in water. The energy content is very small in this fuel yet it drives the electric motor for several hours (try to estimate the chemical energy in 5 milliliters of the methanol solution. Some fuel cells run on methanol like this, which has just one carbon atom for 4 hydrogen atoms \( \text{CH}_3\text{OH} \). Oil or gasoline have far more carbon atoms per hydrogen atom, and this is why they produce so much carbon dioxide as a byproduct of making energy. Moving from these fuels to methanol or natural gas (like methane, \( \text{CH}_4 \)) is a big improvement yet one day
we may run pure hydrogen fuel cell driven automobiles with the hydrogen produced by solar energy.

**E9 THE WORLD’S SIMPLEST ELECTRIC MOTOR: a solar powered motor.**

An electric motor takes a basic force in Nature: the magnetic force, and organizes it to turn a shaft which turns wheels or pumps or fan blades etc. Magnets have two poles, called North and South. The Earth is a huge magnet with similar names attached to its poles. The opposite poles of two magnets are attracted together: South to North; on the other hand the like poles, North to North or South to South, repel each other, which you can feel by trying to bring two bar magnets close to one another. If you mounted one magnet on a spindle so that it could rotate freely, you could push it around without touching it directly, by following it around with a second bar magnet.

The electric motor does just this; only one of the two magnets is made by passing an electrical current through a loop of wire. This may sound different, but a permanent bar magnet has organized its atomic structure so that electrons are spinning around atomic nuclei rather like the electrons going around the coil of wire. Following the spinning magnet (actually electromagnet) around with a second magnet is difficult to do, so the ingenious solution is to switch the current in the coiled wire each time it rotates half-way around: this reverses the poles and so the second magnet doesn’t have to be moved around.

It’s hard to describe but simple when you see it. The electric switching device is called a commutator. In our motor here, however, Eric Lindahl realized that you don’t really need to reverse the magnetic poles each time (which would give two ‘pushes’ per revolution of the wire coil) but if you simply switch off the current during half the rotation of the coil it will work. This is what the solar cell does: it only makes electricity when it can ‘see’ the flood light. So the electromagnet gets a ‘kick’ once each revolution and if the bar magnet is located properly, that will keep the motor going. In the solar powered electrical motor in the lab has two solar cells. Can you figure out how these must be wired up to the wire loop in order to generate ‘kicks’ that act in the same direction as each other.

Electric motors account for a large fraction of our national electricity use (more than \( \frac{1}{2} \)). They will likely soon propel the next generation of automobiles: the hydrogen fuel cell produces electricity, which will be the source of the electricity. Electric currents are described by their voltage, current and resistance. Think of the voltage as analogous to the pressure difference driving water through a pipe. The electrical current is analogous to the flow rate and there is an electrical resistance which would be like friction on the walls of the water pipe. The important relations are: 

\[
\text{voltage difference (V)} = \text{current (I)} \times \text{resistance (R)} \quad \text{(Ohm’s law)} \\
V = I \times R
\]

\[
\text{Power (P)} = I \times V = V^2/R = I^2R
\]
where we have used Ohm’s law. By measuring the voltage and resistance of the solar cell you can work out the power it produces (a few milliwatts…not very much).

How much mechanical power is produced as this little motor spins on its ball-point pen axle? We should have to measure the force times velocity of the spinning motor to know this. But an indirect (and quite accurate) way that is less direct is this get the motor spinning then cut off the light. Time the rate of spin as it slows down. This will give you a good estimate of the power it takes to run the motor at a steady speed. The relationship is

$$\Delta \text{kinetic energy}/\Delta \text{time} = \text{power}.$$ 

Here $\Delta$ means ‘change in’. Estimate the kinetic energy as $\frac{1}{2} m v^2$, where $m$ is mass and $v$ is velocity. This is somewhat tricky for the spinning motor because $v$ is not a constant…it increases from 0 at the center to something large at the rim. Thus the faster moving parts of the motor out near the rim may determine the kinetic energy. By approximating and weighing pieces of the motor on a scale, you can make a stab at the power required to run the motor at various speeds and compare this with the electrical power put out by the solar cell. There are other ways, involving measuring the force it takes to push the motor around, which might be possible to measure (then using the relationship

$$\text{power} = \text{force} \times \text{velocity}$$

which comes from

$$\text{work} = \text{force} \times \text{distance traveled by the ‘forced’ object.}$$

Power is the rate of doing work and velocity is the rate of change of distance.

Like the Stirling engine in E4, this electric motor spins very freely by hand: the magnetic forcing then can easily make it go. This is a property of most kinds of engine.

**Demonstration**

**E10 : BICYCLE POWER: Generating electricity from mechanical energy (which comes from chemical energy)**

Generating electricity with a bicycle. This is an important experience for all of us: a generator from a car is wired up to various electric lights, and you experience the transformation of mechanical energy to electricity (then to light). Recall that the most power that the human body can put out for short times is very approximately 1 horsepower (746 watts). Yet on the bicycle it’s a sweat to light a 100 Watt bulb brightly. Why? Inefficiencies of course. Your mechanical work goes partly into heating up the gears and the generator.

A very efficient long-life bulb shows you how little work you have to do when your electricity goes into light more than heat (the long-life bulb is cool to touch, whereas the normal incandescent bulb is not.

There is the opportunity to run your power through the electric ‘watt meter’ which is the same device that is outside every residence. Learn how to read it!