Class 3. Understanding Conductive Cooling: A Thought Experiment

Write your answers on the exercise sheet

While looking at the global bathymetry maps you will have noticed that mid-ocean ridges or ocean spreading centers are relatively shallow and that the depth of the seafloor increases as you move away from the ridges from about 2-3 km at the ridge to about 5-6 km in old ocean basins. This is a result of the ocean plate (or ocean lithosphere) cooling, becoming more dense and sinking. In the next lecture and the following lab, we will explore some of the physics behind this concept and compare the physical predictions with the observations.

In the ocean basins, the lithosphere cools and thickens because heat is conducted vertically into the ocean. We can understand how the ocean basins cool by following a tall thin block of rock as it moves off axis.

In this exercise we are going to try to understand the basic relationship between the depth of cooling and the age of the block of oceanic plate. We will do this by exploring two physical concepts (heat conduction and heat capacity) and then combine them to obtain a relationship of proportionality between the depth of cooling and the age.

The ocean lithosphere looses heat by vertical heat conduction. This process is described by Fourier’s Law, which is written in terms of a differential equation

\[ q = -k \frac{dT}{dy} \]  

where the variables are as follows

- \( T \) temperature in °C (or Kelvins, K)
- \( y \) distance (of depth) in meters
- \( q \) the heat flux or the amount of heat energy that flows across a unit area of the seafloor in unit time measured in J / m\(^2\)/s
- \( k \) thermal conductivity of the material which measures how well it conducts heat.
  - For aluminum, a good heat conductor \( k = 237 \text{ J} / (\text{s K m}) \)
  - For expanded polystyrene, a good insulator \( k = 0.05 \text{ J} / (\text{s K m}) \)
  - For rocks which are somewhere in between \( k = 1 \) to \( 5 \text{ J} / (\text{s K m}) \)

The quantity \( \frac{dT}{dy} \) is the gradient of temperature.

(a) What is the meaning of the negative sign in equation (1)?
Now if we consider a block of material such as that shown above of unit horizontal cross sectional area and thickness h, that is cold on top (temperature $T_1$), hot on the bottom (temperature $T_2$), and insulated on the sides, we can use equation (1) to express the rate $\bar{q}$ at which heat leaving the box across its top surface of the box

$$\bar{q} = k \frac{T_2 - T_1}{h}$$

(2)

(b) Explain this expression. What has happened to the negative sign that was in equation (1)?

(c) What happens to the value of $\bar{q}$, if we double the thickness of the block but keep the temperature on the top and bottom the same?
Concept 2. Heat Capacity

To understand how quickly a material cools, we not only have to understand how rapidly it looses heat energy, we also have to understand how much heat energy it contains and how that is related to the temperature. This concept is described by a quantity known as the specific heat capacity, $c_p$, which measures the amount of heat energy that must be added to a one kilogram of a material to increase the temperature by one Kelvin (or one degree Celsius).

- Water has a high specific heat capacity, $c_p = 4200 \text{ J} / (\text{kg K})$
- Gold has a low specific heat capacity, $c_p = 130 \text{ J} / (\text{kg K})$
- Rocks have a heat capacity , $c_p = \sim 1000 \text{ J} / (\text{kg K})$

If we multiply the specific heat capacity, $c_p$ by the density, $\rho$ (the number of kilograms per meter cubed) then we get the volumetric heat capacity, $c_p\rho$, which measures the amount of heat energy required to increase the temperature of a meter-cubed volume of material by one degree Celsius.

Now consider our block of material and imagine that the whole block starts out at a high temperature $T_2$. If we want to cool it to a lower temperature $T_1$, we need to remove heat energy. The total amount of heat energy we need to remove is given by

$$ Q = (1 \times 1 \times h)(\rho c_p)(T_2 - T_1) $$

(d) Explain in words why this equation is correct? Think about what each of the expressions in the parentheses represent?
(e) Now if we go back to considering our two blocks. What is the effect of doubling the thickness of the block on the amount of heat that must be removed to cool the temperature from $T_2$ to $T_1$?

Concept 3. The relationship between the depth of cooling and time.

In the above we have developed two quantities

- $\bar{q}$ - the rate at which heat energy is lost from our block
- $Q$ - the total amount of heat energy that must be lost to cool the block

(f) How can we use $Q$ and $\bar{q}$ to obtain an expression for the time, $t$, it takes to cool the block?
(g) Now considering your answers to (c), (e) and (f) how much longer will it take to cool a block that is twice as thick? Explain your answer.

(h) Based on your answer to (g), which of the following three alternatives is the correct relationship between the depth of cooling, $h$ and time of cooling, $t$ (in each expression $\kappa$ is a constant that we will quantify in the next class)

(i) $h \approx \kappa t$
(ii) $h^2 \approx \kappa t$
(iii) $h^3 \approx \kappa t$