Introduction. These are informal notes on the overturning circulations. We think of oceans and atmosphere has having strong lateral circulations…nearly horizontal, plus meridional overturning (up-down, north-south) modes. The two types of circulation interact in many ways, but it is often useful to think of them separately. In previous weeks we have looked at models of the wind-driven gyres in which Sverdrup balance and Rossby waves were important. Indeed, the ‘generalized Sverdrup equation’,

\[ \beta v = f w z \]

governs much ocean dynamics including steady circulation and long, baroclinic Rossby waves. \(v, w\) are north/south and vertical velocities, \(f = 2\Omega \sin \varphi\), \(\beta = 2\Omega / a \cos \varphi\), \(\varphi\) is latitude, \(a\) is Earth’s radius). We have seen that it is nearly a geometric relation expressing the stiffness imparted to the fluid by Earth’s rotation.

Stommel and Arons, in a series of papers in the late 1950s, explored several aspects of the MOC. Whereas only thermodynamics had previously been discussed, they show how PV dynamics can strongly control the circulation, and indeed lead to surprising reversals of flow and strong boundary currents. They split the problem into two parts: from thermodynamics of a 1-dimensional vertical equation for the density they argue that the thermocline is maintained by upwelling from below .. otherwise it would slowly diffuse downward. This balance schematically is

\[ w \rho_z = (\kappa \rho_z)_z \]

where \(\rho\) is density, and \(\kappa\) is vertical diffusivity. We really should be writing conservation equations for temperature, salinity and mass, because density is not what diffuses, but this is an acceptable shorthand. Vertical motion is, in steady state, associated with curvature of the \(\rho(z)\) profile or variations in \(\kappa\) with \(z\). A scale analysis of this equation suggests that the thickness of the thermocline should be \(\delta \sim \kappa / w\). Of course molecular heat diffusion has a value of \(\kappa = 1.4 \times 10^{-7} \text{ m}^2/\text{sec}\) which is so small that the ‘\(\kappa\)’ above has to be interpreted as mixing coefficient due to breaking internal waves or other fine-scale turbulence. Salt involves large ions which diffuse even more slowly: \(\kappa \sim 1.5 \times 10^{-9} \text{ m}^2/\text{sec}\) for salinity. Estimates from ocean microstructure studies suggest mid-ocean values of \(\kappa\) of order \(0.1 \times 10^{-4} \text{ m}^2/\text{sec}\), although the following simple estimate shows that we need more mixing than this.

Munk, in a famous paper called *Abyssal Recipes* (Deep-Sea Res. 1966) took the known sinking rate for the world ocean (totaling roughly 30 Sverdups) and argued that if it returns uniformly to the surface, the one-dimensional heat and salt balance requires a mixing coefficient of order \(10^{-4} \text{ m}^2/\text{sec}\), or ten times greater than widely observed. While it now seems certain that horizontal processes (terms like \(v\partial \rho / \partial y\)) are important and there is conclusive evidence for enhanced mixing near sloping ocean boundaries (where internal wave reflection can enhance shear), e.g., Toole, Polzin & Schmitt, Science 1994. This has led some (e.g., Wunsch & Ferrari, Ann Revs.Fluid Mech 2004)
to conclude that the MOC is ‘driven by wind’ and tides’ rather than buoyancy forcing at the surface (the traditional view); there is by no means agreement on this point.

The Veronis effect. Numerical models of ocean circulation often impose a uniform, constant value for $\kappa$, though a few parameterize $\kappa$ in terms of the current shear, or sometimes prescribe a variation in the vertical. They also can introduce numerical error due to the x-y-z grid that most models employ. Because ocean mixing is very anisotropic (big mixing along isopycnal surfaces, small mixing across them), very different numbers are used for $\kappa_{h,\rho_{xx}}$, say, and $\kappa_{v,\rho_{zz}}$ terms. Typical values for the lateral mixing coefficient are $\kappa_{h} \sim 10^{3} \text{m}^{2}/\text{sec}$ in models whereas vertical mixing coefficient is held down to values of $10^{-4} \text{m}^{2}/\text{sec}$ or so. Consider what will happen in a boundary current where the isopycnal surfaces slope steeply. Here the horizontal mixing will mix temperature and salinity across surfaces of constant density. For many years this effect (named after George Veronis who pointed it out in the mid 1960s, only to see people neglect to correct if for decades) produced the dominant upwelling pathway in ocean circulation models: the upwelling occurred along the western boundary, in the western boundary current, over just a few grid cells. There is no reason to think that this is correct. Models like HIM and MICOM that use density layers rather than horizontal planes as the vertical coordinate surface avoid this problem (although they have other problems). By rotating the diffusivity tensor to recover the anisotropy, along-density-surfaces versus across-density-surfaces), the Veronis effect can be reduced. Other mixing schemes designed to fit this anisotropy are being widely applied (e.g., the ‘Gent-McWilliams’ scheme).

Stommel-Arons model. What Stommel did was to assume that the diffusion and thermocline problems would one day be worked out, but meanwhile they proposed an elegant model for the deep ocean below the thermocline. If upwelling is used to ‘hold the thermocline up’ and prevent it from diffusing downward, then why not assume it to be uniform everywhere as a first simple model? Thus they gave birth to the single-layer model of deep source-driven circulations. The fluid sources are meant to replace the sinking regions which send water downward. The model can be employed on the spherical Earth, or as a GFD lab model with depth variations providing the mean PV gradient rather than the spherical $\beta$ effect.

We worked out a simple flow in class in a square basin with a source in the northwest corner, and a lab model in which the depth varied linearly from north to south. In this case the upwelling of the free surface with imposed velocity $W_{0}$, together with the PV equation in the form $Dh/ Dt = 0$ (h is the fluid column height), leads to $v \partial h/ \partial y = W_{0}$ If the slope of the bottom is $\alpha$, this gives simply

$$v = W_{0}/ \alpha$$

Note we are conserving $f/h$ following the fluid motion, under the assumption that the relative vorticity, $\zeta$, is negligibly small in mid-ocean. Mass conservation is a separate constraint of the form $u_{x} + v_{y} + w_{z} = 0$ for an incompressible fluid. But here we have seen that $v_{y}=0$ and the PV balance insists that $w_{z} = 0$ so it follows that $u_{x} = 0$. In the presence of coastal boundaries, this means that $u = 0$ itself.
So, in mid-ocean, the circulation is northward at a uniform velocity. This is surprising because we placed the source of fluid in the north, and the fluid circulates towards it. Stommel & Arons solve this paradox by arguing that western boundary currents will develop to carry fluid southward away from the source, and eastward along the southern boundary. There is ample evidence for western boundary currents by now, including Rossby wave propagation ideas. In these boundary currents friction terms become large enough to allow fluid to move rapidly across mean PV contours, diffusing away their anomalous PV. We will see later that in fact nonlinear terms may also be important, implying greater conservation of PV in boundary currents than this model would suggest.

In other papers (for example the 3 page note by Stommel and Arons 1958, handed out in class) this simple idea, that the oceans should be flowing poleward in most of the world ocean, led to a globally complete model of the MOC. The boundary currents required to conserve mass as well as PV were the most observable of the predictions, and indeed deep western boundary currents are found widely round the world.

The SAF paper continues with source-sink pairs placed on the eastern boundary, generating flows that involve zonal jets of current that connect through western boundary currents.

Recirculation gyres: vorticity forcing. After this model was constructed we have found that sources of fluid can be sources of PV as well as sources of mass. This means that a source injects fluid with a specific PV that may or may not fit in with the mean f/h of the basin. Very often this causes strong gyres to form, and extend westward as β-plumes. These gyres deflect the mass-flow away from simple straight paths from source to sink. They are a feature of the real ocean, and make the ‘conveyor belt’ seem a poor choice of words for the MOC.

In his PhD thesis, Bob Hallberg (Hallberg & Rhines JPO 1996) demonstrated this recirculation effect in a 2-layer model of an ocean basin driven by upwelling/downwelling through the thermocline. This we called ‘heating’ or ‘cooling’ because it converts water from one density to another, in a specified small region. It is like a Stommel-Arons model but with an upper ocean layer fully present. The ‘cooling’ function chosen is a small circle of cooling in the subpolar regions. The β-plume gyre is strongly evident, and links the mid-ocean forcing region to the western boundary. From there, western boundary currents carry the circulation equatorward, and then to the eastern boundary, then into Rossby waves which finally fill out the interior Stommel-Arons poleward flow. This is in the same spirit of the Kelvin waves and Rossby waves that Kawase (JPO 1987) used to explain the spin-up of the Stommel-Arons circulation. We added basin topography with continental slopes, which made much better ‘waveguides’ for the boundary currents. The relevant wave modes become topographic Rossby waves rather than Kelvin waves, and the β-plume refracts equatorward smoothly rather than making a difficult transition into a Kelvin wave at the western boundary.
The MOC is often portrayed as being slow to react or change. The ‘flushing time’, that is the time it takes fluid to pass through the entire deep ocean before emerging again at the surface, is estimated to be thousands of years (using an crude estimate of global sinking rates, say 20 Sverdrups, divided into the volume of the world ocean). However, boundary waves are a fast response to changes in the high-latitude sinking and indeed, the recirculating gyres can respond quickly. Thus it is possible that the MOC itself lurches and changes rather quickly.

Recirculation gyres: closed \( f/h \) contours. Planetary ‘islands’ form in deep ocean basins and around islands and topographic rises where \( h \) varies enough to overcome the gradual variation of \( f \), in the \( f/h \) PV field. If we try to apply Stommel-Arons’ solution in such regions, it fails. The idea that fluid can move upslope to accommodate the enforced upwelling \( W_0 \) is fine on a simple slope (as above) but if we apply it where \( f/h \) has a local maximum or minimum it would require a sink or source of fluid at the extremum. Only if \( W_0 \) were to vary in space such that its line integral around each closed \( f/h \) contour were to vanish, could the solution work.

Instead, other terms in the PV have to become important is such regions, for example friction or time-dependence or nonlinearity. Strong gyres are created, running around the closed \( f/h \) contours, which are ‘free pathways’ for circulation (see Straub & Rhines, J.Marine Res. 1989).

Generally, the poleward interior circulation has not been observed even with the best ‘current meter’, the chemical tracer fields. The intense recirculations described above are part of the reason. But another key reason is that the upwelling velocity may not be anything like uniform in space. We now believe that much global upwelling occurs in the Southern Ocean where the polar and subpolar fronts create nice, steep isopycnal surfaces along which water can rise with minimal buoyancy forces. The deep water must somewhere mix in order to reach the sea surface but this may happen in special regions. It is disappointing that we do not know conclusively the pattern of upwelling from the deep ocean. Chemical tracers and biological distributions may give us the answer.

The global climate heat-engine. Together the oceans and atmosphere are a ‘heat- and fresh-water engine’. Solar radiation is the source of most of the energy of the general circulation, and it arrives at low latitude more strongly than at high latitude, where there is net outward radiation. The contrast is not as great, however as simple radiation models would suggest. The figure below shows the latitude variation of surface temperature of a ‘bare Earth’ with no atmosphere or ocean, and of a one-pane-of-glass greenhouse Earth. Both show much too much variation in temperature with latitude. The observed mean variation (also plotted) is the result of circulation, which carries warmth and water vapor poleward.
Variation in Earth’s surface temperature according to a simple radiation balance (no atmosphere, ‘Bare Earth’) which is too cold and has too much latitude variation, a simple greenhouse Earth, and the observed hemispheric average temperature. (K.Carslaw, Univ. of Leeds).

**Meridional energy transport.** Global meridional heat transport, which is known from ERBE satellite radiation measurements, has a smooth, nearly sinusoidal variation with latitude (bold solid curve above). Indeed, thermal energy is carried from the tropics...
poleward. Note that it is the slope of the curve that is the heat gain or loss. The total transport divides roughly equally into 3 modes:

1. (dry static energy) atmosphere \(\sim 3 \text{ pW} (= 3 \times 10^{15} \text{ Watts})\) (grey curve above)
2. (sensible heat) ocean \(\sim 2.5 \text{ pW}\) (thin solid curve above)
3. water /latent heat: a joint atmosphere/ocean mode \(\sim 2.2 \text{ pW} = 1.0 \text{ Sv} \) (1 megatonne per sec) (dashed curve above)

The three modes of poleward transport are comparable in amplitude, and distinct in character (sensible heat flux divergence focused in tropics, latent heat flux divergence focus in the subtropics). Note particularly the slope of the latent heat flux curve, which is equal to the evaporation minus precipitation through the sea surface \((E - P)\). [A small correction occurs due to river flows on land.]

Thus although the atmosphere seems to dominate the poleward flux of thermal energy at high latitude, the ocean is contributing in many ways. It provides virtually all the moisture and most of the sensible heat to drive the atmospheric MOC. It affects the atmospheric MOC by being a warm, moist lower boundary condition which facilitates heat transport to very high latitude. Through storm-track dynamics it materially affects the atmospheric MOC. The storm tracks are region of intense cyclonic systems in the northern Atlantic and northeastern Pacific sectors. These regions dominate the poleward flux, if one looks at its east-west structure rather than just the zonal average shown above. The ocean’s cryosphere…sea-ice affects the picture greatly, through albedo effects and through ‘insulating’ the atmosphere from the ocean, preventing strong heat flux or \(E - P\).

Let us look at some of the numbers. Poleward oceanic heat transport can be written
\[
\int \rho C_p \theta v \, dz
\]
where \(\rho\) is density \((\text{kg/m}^3)\), \(C_p\) is specific heat capacity, 3986 Joules/(kg K) at atmospheric pressure, 35 ppt salinity and 10C temperature, \(\theta\) is potential temperature and \(v\) is northward velocity. The vertical integral is then integrated zonally to cover the whole ocean. Meteorologists account for compressibility of the atmosphere by writing for the sensible heat flux (‘dry static energy flux’) written as
\[
\int (C_p T + g z) v \, dp
\]
where \(p\) is pressure. \(C_p\) for dry air is about 1005 Joules/(kg K) (Gill p.42). The atmospheric latent heat flux is
\[
\int L q v \, dp
\]
where \(L\) is the latent heat of vaporization of water (about \(2.5 \times 10^6\) Joules/kg) and \(q\) is the specific humidity (kg water/kg air).

Suppose the Atlantic Ocean has an MOC transport of 16 Sverdrups (typical of current estimates). If the difference in potential temperature between the northward flowing warm waters and the southward flowing cold waters is 25K (Kelvins), then the heat transported will be 4000 Joules/(kg K) x 20 K x 16 x \(10^9\) kg/sec, or 1.28 x \(10^{15}\) Joules/sec (\(\equiv\)Watts). This is 1.28 petaWatts, close to the observed value for the Atlantic at 26\(^0\)N latitude.

Now look at the amplitude of the meridional latent heat flux in the figure above. It peaks near 2 pW. How much moisture is being transported? The latent heat of vaporization is about \(2.25 \times 10^6\) Joules/kg. So \(2 \times 10^{15}\) Joules/sec divided by 2.25
Joules/kg gives $0.9 \times 10^9$ kg/sec. That is, 0.9 Sverdrups of water vapor moving poleward. One Sverdrup is a kind of scale magnitude for the hydrologic cycle in this sense (although if you add up all the precipitation globally it is more like 17 Sverdrups). Note that if you remove all the moisture from the air above your head it will be only about 3 cm deep...the humidity is low on average. Yet this small amount of water vapor carries tremendously much thermal energy...water vapor is ‘stealth’ heat. It is released where cloud droplets and rain and snow form.

A similar calculation can be made to relate the atmospheric water vapor flux to the observed salinity difference between the warm northward flowing waters of the MOC and the cold, deep southward flowing waters. We observe a salinity change $\delta S$ of typically from 35.5 – 36.0 (shallow northward flow) down to 34.9 in the deep southward flow or roughly 1 ppt change out of 35. Salinity, $S$, is defined as mass of salt (kg)/(mass of salt + mass of freshwater).

The details are in the footnote below. The result is that the evaporation is $E = \frac{1}{2} \left( \frac{\delta S}{S} \right) F_{SW}$, where $F_{SW} = 16$ Sverdrups. Thus we predict that the evaporation, and hence the poleward flux of water vapor should be roughly

$$\frac{1}{2} 2.8 \times 10^{-2} \times 16 \text{ Sverdrups} = 0.23 \text{ Sverdrups of moisture flux.}$$

This is a fairly convincing estimate for the Atlantic sector (the entire zonally integrated world contributing about 0.8 Sverdrups at subtropical latitude, as seen in the figure above)

It is important to bind together these estimates, using every independent observation that one can, in trying to get a true picture of the MOC.

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We equate the water lost to evaporation ($E$, in kg/sec) in the subtropical oceans to the difference in water transport by the upper and deep branches of the circulation (ignoring land effects). It is slightly tricky. The FW (fresh water in kg) flux in kg/sec is $F_{FW} = C_{FW} F_{SW}$ where $C_{FW}$ is the concentration of fresh water (kg of fresh water/ kg of seawater) and $F_{SW}$ is the flux of seawater (FW + salt). So we have

$$E = F_{FW} \text{ (deep southward)} - F_{FW} \text{ (shallow northward)}$$

$$= C_{FW} F_{SW} \text{ (deep)} - C_{FW} F_{SW} \text{ (shallow)}$$

$$= C_{FW} F_{SW} \text{ (deep)} - (C_{FW} \text{ (deep)} + \delta C_{FW})(F_{SW} \text{ (deep)} - E)$$

$$= C_{FW} F_{SW} \text{ (deep)} - (C_{FW} \text{ (deep)} - \frac{\delta S}{S})(F_{SW} \text{ (deep)} - E)$$

where we use the fact that, from the definition of salinity, $S$, the salinity change and the fresh-water change are complements of one another: $\delta S/S = -\delta C_{FW} \approx -\delta W/FW$ and we restrict to $\delta S/S \ll 1$.

So the result is that $E = \frac{1}{2} \left( \frac{\delta S}{S} \right) F_{SW} \text{ (shallow)}$