Note on the vertical structure of the circulation. 17v07

We argued that for there to be circulation in the ocean beneath the mixed layer, the PV contours need to be greatly deformed by the circulation itself and perhaps ‘gyre like’. Otherwise they will just be latitude circles and will bump into continents, both east and west.

The geometric argument uses the large-scale PV, $\frac{PV}{f/h}$ where $h$ is the thickness vertically of an isopycnal layer. We showed that we needed $\frac{f}{h} \sim \beta$

Another, quick way to see the result is to use the quasi-geostrophic PV expression,

$$PV_{QG} = \nabla^2 \psi + (\frac{f_0^2}{N^2} \psi_y)_{x} + f(y)$$

If we ignore the relative vorticity, $\zeta \equiv \nabla^2 \psi$, the PV equation is

$$\left(\frac{v \partial}{\partial y} + u \frac{\partial}{\partial x}\right)\left(\frac{f_0^2}{N^2} \psi_y + f(y)\right) = 0$$

the 2d and third terms will be comparable if

$$\frac{f_0^2}{N^2} \psi_{yy} \sim \beta$$

But $\psi_y = -u$, so this is

$$\frac{f_0^2}{N^2} u_{yy} \sim \beta$$

The scale analysis of this gives the vertical length scale of the gyre as

$$\delta \sim \frac{f_0}{N(U/\beta)^{1/2}}$$

as we derived in class. Putting numbers in we indeed get a fairly large number for the vertical extent of the wind-driven gyres...I had forgotten how deep the predicted penetration actually is. If $N= 3 \times 10^{-3}$, $f = 10^{-4}$ (so that $f/N = 0.033$) then a choice $U \sim 0.02$ m/sec (2 cm/sec) and $\beta \sim 2 \times 10^{11}$ m$^{-1}$sec$^{-1}$ gives

$$\delta \sim 1000m$$

So the wind-driven gyres can penetrate that deep where the current speed averaged over 1000m $\sim 0.02$ m/sec. Surface currents tend to be stronger than this but in mid-ocean they tend to die down at greater depth. We had used $U \sim 0.1$ m/sec which is more characteristic of the separated boundary jets, not the quieter midocean gyre circulation (once we average over the depth suggested by $\delta$).

Incidentally, this same argument predicts that the Rossby radius for such a gyre circulation should be simply

$$\lambda \sim (U/\beta)^{1/2}$$

because $\lambda = N\delta/f$ by definition, for motions with height-scale $\delta$.

Another result of interest is that, if Sverdrup dynamics rules, we can relate $U$ to the wind-stress curl. The Sverdrup relation gives $\delta U \sim \nabla \times \tau / \beta$ hence
\[ \delta^3 = f_0^2 / N^2 \nabla x \tau / \beta^2 \]

We might want to allow the zonal and meridional velocities in the general circulation to be different, which would include a factor \( U/V \sim \) aspect ratio of the gyre.

Remember that the PV maps we have made (McDowell et al, Keffer...) are just doing this same thing more exactly, and as you see they can show PV greatly altered from latitude circles, at quite great depth. Look at Keffer's figures below: the \( \sigma_0 = 27.4 \) surface is about 1200m deep in the N Pacific, and the PV is thoroughly altered by the circulation even at this depth; it is almost uniform from 25N to 55N despite the large change in \( f \) over this interval.
Meridional plots of the PV show the f(y) ‘hillside’ cut by ‘plateaus’ of gyre PV. Talley (JPO 1988) shows more detail of the Pacific PV fields: here are some meridional cuts:

Meridional plots of the PV show the f(y) ‘hillside’ cut by ‘plateaus’ of gyre PV.
Below (her Fig. 8) you see a section showing the regions in which $\frac{\partial}{\partial y}$ (PV) is roughly $\beta$, or very different from $\beta$, as we have suggested:
Look at the way the outcrops and the PV interact:
It is remarkable how deep the PV signal penetrates. Note that no outcrops exist for densities greater than $\sigma_0 = 26.6$ or so:
Rick Williams has continued this work, looking at the deep PV signal (JPO 2002, JGR 2002).