GFD - 2   Spring 2004   P.B. Rhines   Quiz 1  13 May 2004

Please return the quiz to me by beginning of class, 3.30 pm Friday   (319 Ocean Sciences). You may use class notes and Gill’s text as reference.

1. Short answers

   a. The Gulf Stream flowing eastward from the US has a Coriolis force to its right, with sea surface tilting ...... and isopycnal surfaces tilting ...... to the north. (up or down).

   b. Use scale analysis to show that the Ekman layer thickness is roughly equal to the depth to which simple viscous diffusion of momentum (obeying $u_t = \nu u_z$) will penetrate in one day (if we switch on the wind-stress, this is how the Ekman layer develops). (Can you improve my estimate of ‘one day’, also?). $\nu$ is the kinematic viscosity coefficient.

2. On the attached maps of wind-stress components ($\tau_x$, $\tau_y$) over the world ocean draw arrows indicating the Ekman transport and indicate regions of Ekman driven upwelling and downwelling, $w_E$. Also indicate where the magnitude of the $w_E$ is particularly large. Note, the Ekman pumping formula describes $w_E$ in regions not near side boundaries, so you will want to include in your thinking the vertical velocity at a coastal boundary, due to horizontal Ekman transport. Note the cgs unit of stress is 1 dyne cm$^{-2}$, which is a typical wind stress; this is equal to 0.1 Pascal or 0.1 Newton m$^{-2}$.

3. We described the classic theory of the wind-driven circulation based on a 1-layer model and its barotropic potential vorticity, PV. We then introduced the baroclinic PV for a density-stratified ocean, which is conserved following the fluid on each potential-density surface (except for dissipation/external forcing effects). The circulation of the stratified oceans decreases as one moves downward, in thermal-wind balance. In order that gyres of general circulation exist, maps of PV contours must be roughly in the shape of the gyre, rather than being dominated by the planetary vorticity, $f(y)$. If this is true, the contributions to $\nabla q$ from vortex stretching (variation of thickness of isopycnal layers) and from $\beta$ must be comparable, where

   \[ q = \left( \frac{f}{\rho_0} \right) \frac{\partial \rho}{\partial z} \]

   is the large-scale PV.

   *write down the two terms in the expression for $\partial q / \partial y$

   *if the two terms are equal in size find an expression for the vertical shear of the east-west circulation, $\partial u / \partial z$ in terms of $f$, $\beta$ and buoyancy frequency $N$. 
• Use scale analysis on this expression to find the depth to which the wind-driven circulation penetrates, \(d\), in terms of the typical horizontal velocity \(U\), \(f\), \(\beta\), \(N\). This is the ‘thickness’ of the gyres of general circulation. [Here \(\rho_0 = \text{const}\), and \(\rho\) is density, in the Boussinesq approximation.]

• What parameter has to be small in order to neglect vertical relative vorticity, \(\zeta\) in the expression above for potential vorticity, \(q\)?

4. Consider standing (stationary) Rossby waves generated in uniform eastward flow (for example by flow over a mountain). We looked at the case for barotropic Rossby waves (no vertical variation of velocity). Now include a free surface term in the Rossby wave equation, so that it becomes

\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) (\nabla^2 \eta - \lambda^2 \eta) + \beta \frac{\partial \eta}{\partial x} = 0
\]

where \(U\) is a positive constant. \(\eta\) is the free-surface displacement, but it could as well be the geostrophic streamfunction \(\psi\) or pressure \(p\). Now set the \(\partial / \partial t\) terms equal to zero and look for wave solutions \(\exp(ikx + i\ell y)\) of the remaining equation, finding the dispersion relation involving the \(x\)- and \(y\)- wavenumbers \(k\) and \(\ell\).

• Draw the curve in \((k, \ell)\) space showing wavenumbers possible for a given \(U\).

• Describe the new effects (like wavelengths etc.) due to the term involving \(\lambda\) (comparing with the case we looked at, \(\lambda = 0\)).

• Suppose that we are looking at baroclinic Rossby waves with a single vertical wavenumber, \(m = \pi/4000 \text{ m}^{-1}\) (a single vertical mode). For realistic values \(\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{sec}^{-1}\), \(f = 10^{-4}\) and \(N = 15f\), what is fastest possible horizontal group velocity of the long, non-dispersive Rossby waves in this mode (consider also its dependence on latitude)?
Fig. 2. Climatological annual average wind stress: (a) smoothed $\tau^{(a)}$, this calculation; (b) smoothed $\tau^{(b)}$, this calculation; (c) difference between (a) and HR $\tau^{(a)}$; (d) difference between (b) and HR $\tau^{(b)}$. 

SMOOTHED ANNUAL AVERAGE TAUX (CI = 0.2 DY/CM$^2$)

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