1. Two-Box Model for POC flux

Use Broecker’s two-box model and the concentrations listed below to answer the following questions. Note that "B" is the particle matter that sinks out of the surface layer. Use a river water flux ($V_{river}$) of $3.7 \times 10^{16}$ L / yr, and don't worry about the ocean exchange of C with the atmosphere. Mean river and ocean concentrations in moles m$^{-3}$ are given below. Remember that $V_{mix} = 30 V_{river}$.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$C_{river}$</th>
<th>$C_{surf}$</th>
<th>$C_{deep}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.0013</td>
<td>$0.02 \times 10^{-3}$</td>
<td>0.0023</td>
</tr>
<tr>
<td>NO$_3$</td>
<td>0.030</td>
<td>$0.03 \times 10^{-3}$</td>
<td>0.030</td>
</tr>
<tr>
<td>DIC</td>
<td>1.2</td>
<td>1.95</td>
<td>2.30</td>
</tr>
<tr>
<td>Ca</td>
<td>0.36</td>
<td>10.20</td>
<td>10.30</td>
</tr>
</tbody>
</table>

A. Draw the two-box model showing all inputs, outputs and reservoir concentrations for one constituent (P, NO$_3$, DIC, or Ca).

\[ [Ca]_{surf} = 10.20 \text{ mol/m}^3 \]
\[ [Ca]_{deep} = 10.30 \text{ mol/m}^3 \]
B. Calculate the particulate carbon flux to the deep sea. How does it compare to the fossil fuel CO₂ production rate of 5.4 Gt C yr⁻¹ (5.4 x 10¹⁵ g C yr⁻¹)?

- Particulate carbon flux = inorganic and organic carbon so we use DIC as a “tracer”. Note that if we wanted to figure out the inorganic carbon particulate flux then we would use Ca as our tracer since Ca is exported in CaCO₃ particles, the inorganic fraction of the particulate carbon flux.
- To calculate the flux of particulate carbon we assume steady state in the surface ocean box so that inputs equal the outputs.
- To get the correct units of mol/yr we need to convert liters/yr to m³/yr in our calculation.

\[ V_{RCR} + V_{mixC_{Deep}} = V_{mixC_{surf}} + B \]

\[ B = V_{RCR} + V_{mixC_{Deep}} - V_{mixC_{surf}} \]

\[ B = (3.7 \times 10^{16} \text{ L/yr}) \times (1.2 \text{ mol DIC/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr})\times (2.3 \text{ mol DIC/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (1.95 \text{ mol DIC/m³})(m³/1000L) \]

\[ B = 4.329 \times 10^{14} \text{ mol DIC/yr} \]

Now convert DIC to carbon and get the units into g/yr.

\[ B = (4.329 \times 10^{14} \text{ mol DIC/yr}) \times (1 \text{ mol C/1 mol DIC}) \times (12 \text{ g C/mol C}) = 5.195 \times 10^{15} \text{ g C/yr} \]

\[ B = 5.2 \text{ Gt C/yr} \]

The calculated particulate carbon flux is smaller than the fossil fuel production rate of 5.4 Gt C/yr by 3.7%.


\[ B_C = (4.329 \times 10^{14} \text{ mol DIC/yr}) \times (1 \text{ mol C/1 mol DIC}) = 4.329 \times 10^{14} \text{ mol C/yr} \]

\[ B_N = (3.7 \times 10^{16} \text{ L/yr}) \times (0.03 \text{ mol NO}_3/m³)(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (0.03 \text{ mol NO}_3/m³)(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (0.03x10^{-3} \text{ mol NO}_3/m³)(m³/1000L) \]

\[ B_N = 3.438 \times 10^{13} \text{ mol NO}_3/yr \]

\[ B_P = (3.7 \times 10^{16} \text{ L/yr}) \times (0.0013 \text{ mol P/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (0.0023 \text{ mol P/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (0.02x10^{-3} \text{ mol P/m³})(m³/1000L) \]

\[ B_P = 2.579 \times 10^{12} \text{ mol P/yr} \]

\[ B_{Ca} = (3.7 \times 10^{16} \text{ L/yr}) \times (0.36 \text{ mol Ca/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (10.30 \text{ mol Ca/m³})(m³/1000L) + 30 \times (3.7 \times 10^{16} \text{ L/yr}) \times (10.20 \text{ mol Ca/m³})(m³/1000L) \]

\[ B_{Ca} = 1.213 \times 10^{14} \text{ mol Ca/yr} \]
To get the molar ratio of P:N:C:Ca divide everything by mol P/yr.

\[
\begin{align*}
P/P &= (2.579 \times 10^{12} \text{ mol P/yr})/(2.579 \times 10^{12} \text{ mol P/yr}) = 1 \\
N/P &= (3.438 \times 10^{13} \text{ mol NO}_3\text{/yr})/(2.579 \times 10^{12} \text{ mol P/yr}) = 13.3 \\
C/P &= (4.329 \times 10^{14} \text{ mol C/yr})/(2.579 \times 10^{12} \text{ mol P/yr}) = 167.9 \\
Ca/P &= (1.213 \times 10^{14} \text{ mol Ca/yr})/(2.579 \times 10^{12} \text{ mol P/yr}) = 47
\end{align*}
\]

<table>
<thead>
<tr>
<th>My value</th>
<th>Broecker and Peng (1982)</th>
</tr>
</thead>
</table>

The molar ratios calculated here are slightly different than those cited by Broecker and Peng. The N:P ratio is lower and the C:P and Ca:P ratios are higher. This suggests that in order for our box model to be in steady state, the export of nitrate must be lower and the export of calcium and carbon must be greater than what Broecker and Peng found from their research.
2. Radioactive Decay

$^{210}$Po is a very toxic isotope (about 106 times more toxic than cyanide). The main hazard is its intense alpha radiation which, if ingested, does serious damage to internal tissues. Imagine that you are an undercover spy and you want to poison some critics of your government by slipping some $^{210}$Po into their margaritas. The maximum safe body burden of $^{210}$Po is ~1000 Bq = 16.7 dpm. To make sure your deadly poison works you want to increase the body burden by >10 times to >167 dpm.

Hint: 1 Becquerel (Bq) = 1 dps = 0.016 dpm

a) Your chemical lab knows how to separate $^{210}$Po from $^{210}$Pb. You just have to get them the material. You have 200 dpm of pure $^{210}$Pb. How long do you have to let the $^{210}$Pb decay so that you have almost (95%) 200 dpm of $^{210}$Po? (10 pts)

We know from the above section that these species are in secular equilibrium. So we can apply our knowledge of “grow-in” curve: Starting with a reservoir of “parent” material with activity $A_A$, the activity of the daughter material $A_B$ will steadily increase.

At time $= 1$ daughter half-life, or $t_{1/2_B}$, the daughter activity will have “grown in” to 50% of the activity of the parent.
At time $= 5$ daughter half-lives, or $5 \times t_{1/2_B}$, the daughter activity will have “grown in” to 95% of the activity of the parent.

From above, we know the half-life of the daughter ($^{210}$Po) is 138 days. So to reach an activity $A_B$ of 95%, we would need to wait 5 half-lives of $^{210}$Po:

138 days $\times 5 = 690$ days, or almost two years. (Wow, pretty slow spy operation!)

We could also use the equation in the course pack (p. 139) to determine this result:

$$A_B = A_{A,0}(1 - e^{-0.693t / t_{1/2}})$$
$$A_B / A_{A,0} = 0.95 = 1 - e^{-0.693t / t_{1/2}}$$
$$0.95 - 1 = e^{-0.693t / t_{1/2}}$$
$$0.05 = e^{-0.693t / t_{1/2}}$$
$$\ln(0.05) = -0.693t / t_{1/2}$$
$$t = -(\ln(0.05) \times t_{1/2}) / 0.693$$
$$t = -(2.996 \times 138 \text{ days}) / 0.693 = 597 \text{ days}$$ (so, the rule about 5 half-lives is an approximation)
b) Your lab chemists do their job well and now you have a sugar cube containing 200 dpm of $^{210}$Po. You get ready to meet your victim in a bar in Fremont but suddenly he is out of the country (gone to Britain) for 138 days. Finally you meet and while his back is turned you slip the sugar cube into his drink. Will your poison work?? (10 pts)

The victim managed to stay out of the country for one entire half-life of $^{210}$Po -- curses! Now the activity is 1/2 of what it was originally, so the sugar cube’s $^{210}$Po activity is:

$$200 \text{ dpm} / 2 = 100 \text{ dpm}$$

According to the above info, this isn’t enough to increase the body burden by >10x, or 167 dpm, so we can’t be sure the poison will work. However, it’s a pretty hefty dose, so your victim will definitely be impaired, at least.