OCN/ESS/ATM S 588 Global Carbon Cycle and Greenhouse Gases
Problem Set 2 - due in class January 31, 2004

Four Box Ocean/Atmosphere Model
The goal of this problem set is to determine how the fCO2 of the atmosphere depends on fHD (the mixing between the deep and high latitude boxes) and PH (the particle flux from the high latitude box). We will be calculating steady state solutions, not transitions from one state to another. Use the function co3eq to determine the fCO2 of the water.

Assume:
1. Use the same volumes, areas, depths, temperatures, salinities and total phosphate, alkalinity and carbon contents of the ocean + atmosphere system as in Table 2 of Toggweiler and Sarmiento.
2. T = 20 Sv (10^6 m^3/s), the overturning circulation is fixed for all parts of the problem.
4. fLD = fLH = 0. No mixing between the low latitude box and either the deep box or high latitude boxes, only T moves water between these boxes.
5. GL = GH = 3 m/d The gas exchange coefficient is constant and the same rate for both low and high latitude boxes. The flux due to gas exchange = G * KH * Area * (pCO2atm - pCO2water). KH is the solubility of CO2 in seawater. KHLow = 0.0312 mol / kg / atm, KHHigh = 0.0572 mol / kg / atm.
6. PL = PH for today's ocean (parts A-D) in moles P / m^2 / yr.
7. 1 L = 1.023 kg of seawater

A) We suggest that you solve this problem by using matrices to solve sets of simultaneous equations. Write out the sets of equations for PO4, Alkalinity and Carbon. Then transform those sets of simultaneous equations to matrix form, so that they can be solved easily in MatLab.
1. For the PO4 equations, you will need to write 2 equations, one each for the mass balance of high and low latitude ocean boxes. Your knowns are the phosphate concentrations of the three ocean boxes and T. Use Toggweiler and Sarmiento’s pre-industrial simulation PO4L, PO4H and PO4D values from Figure 7. Your unknowns are PH (=PL) and fHD.
2. For the Alkalinity equations, you will need to write 3 equations, one mass balance equation each for high and low latitude ocean boxes and one conservation equation. Your knowns are the total Alkalinity content of the ocean, the volumes of the different ocean boxes, T, PL, PH, fHD and the ratio of Alk:PO4. Your unknowns are the Alkalinites of the three ocean boxes, AlkL, AlkH, and AlkD.
3. For the total carbon equations, you will need to write 4 equations: one mass balance equation each for 3 of the 4 ocean/atmosphere boxes and one conservation equation. Your knowns are the total carbon content of the ocean + atmosphere, the volumes of the different ocean/atmosphere boxes, the areas of the two ocean boxes, T, PL, PH, fHD, the gas exchange coefficient (G), the solubility of CO2 (KH), the ratio of DIC:PO4, and the fCO2 of the low and high latitude boxes (fCO2L and fCO2H). Your unknowns are the DIC’s of the three ocean boxes, DICL, DICH, DICD and the fCO2 of the atmosphere.
B) Using your 2x2 PO4 matrix from part A, calculate PH (the particle flux from the high latitude box), PL (the particle flux from the low latitude box, = PH) and fHD (the mixing between the high latitude and deep boxes).

C) Using your 3x3 Alkalinity matrix from part A and the Pt, PH and fHD you determined in part B, calculate the Alkalinity of the three ocean boxes.

D) Using your 4x4 Carbon matrix from part A and the Pt, PH and fHD you determined in part B, calculate the DIC of the three ocean boxes and the fCO2 of the atmosphere. You will have to use an iterative scheme to do this. You may either use the scheme in the Appendix of Toggweiler and Sarmiento or the one detailed below:

1. Guess initial values for DICL and DICH.
2. Determine fCO2L and fCO2H for initial values.
3. Solve 4x4 Carbon matrix for DIC in the three ocean boxes and fCO2atm.
4. Check whether (new DICL - old DICL < 0.002 \(\mu\text{mol/kg}\)) and (new DICH – old DICH < 0.0005 \(\mu\text{mol/kg}\)). If so, you're done! If not, continue with step 5.
5. Increment DICL. (new DICL = old DICL + (new DICL - old DICL)/100)
6. Increment DICH. (new DICH = old DICH + (new DICH - old DICH)/5)
7. Calculate new fCO2L and fCO2H. Go back to step 4.

An example script is on page 3!

E) Determine the effect of high latitude convection and biological productivity on the atmospheric CO2 concentration. Vary fHD leaving PH and Pt constant, and then redo parts C and D with the new mixing rate to determine the fCO2atm. Next vary PH, leaving fHD and Pt constant, and determine the fCO2atm. Plot your results and comment on the trends you obtain.

**What you'll turn in:**
The values you calculate for PH, fHD, Alkalinity, and DIC.
Figures showing the trend of varying fHD and PH.
Email Lia your MatLab script (m file), including commented equations.
Here’s an example of how you might write this iterative script (you don’t have to use this way, it’s just something that made sense to me):

```matlab
% Initial DIC_high and DIC_low guesses
dic_high_guess = 2100; % units umol/kg
dic_low_guess = 1900;  % units umol/kg
% Calculate fCO2_high and fCO2_low using co3eq
fCO2high_guess= co3eq2(temp_h,sal,0,alkh,dic_high_guess); % units atm
fCO2low_guess= co3eq2(temp_l,sal,0,alkl,dic_low_guess); % units atm

% DIC matrix. Equations are for the deep, high, low ocean boxes and
% conservation of mass, solution X_DIC = A_DIC\B_DIC
WRITE YOUR MATRICES HERE
X_DIC = A_DIC\B_DIC;

    dic_deep = X_DIC(1); % units umol/kg
dic_high_out = X_DIC(2); % units umol/kg
dic_low_out = X_DIC(3); % units umol/kg
fCO2a = X_DIC(4); % units atm

% check if guesses were close enough and iteratively solve the matrix
until they are.
while abs(dic_high_out-dic_high_guess)>0.0005|abs(dic_low_out-
dic_low_guess)>0.002
    dic_high_guess = dic_high_guess + (dic_high_out -
dic_high_guess)/5; %units umol/kg
    % so if our guess is far off, we're make a bigger correction than
if our guess was close.
    dic_low_guess = dic_low_guess + (dic_low_out - dic_low_guess)/100;
    % use new dic values to solve for fCO2
    fCO2high_guess= co3eq2(temp_h,sal,0,alkh,dic_high_guess);%units atm
    fCO2low_guess= co3eq2(temp_l,sal,0,alkl,dic_low_guess); % units atm

    % now reiterate matrix solution
WRITE YOUR MATRICES HERE
X_DIC = A_DIC\B_DIC;

    dic_deep = X_DIC(1); % units umol/kg
dic_high_out = X_DIC(2); % units umol/kg
dic_low_out = X_DIC(3); % units umol/kg
fCO2a = X_DIC(4); % units atm
end
dic_high = dic_high_out;
dic_low = dic_low_out;
fCO2a_ppm = fCO2a*10^6
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\[ g = \text{air water flux (mol yr}^{-1}\text{)} = G K S (p_{\text{CO}_2,a} - p_{\text{CO}_2,w}) \]
\[ G = \text{gas exchange mass transfer coefficient (m yr}^{-1}\text{)} \]
\[ KH = \text{solubility (mol m}^{-3}\text{atm}^{-1}\text{)} \]
\[ S = \text{surface area (m}^{-2}\text{)} \]
\[ p_{\text{CO}_2} = \text{CO}_2 \text{ partial pressure (atm)} \]
\[ C = \text{concentration (mol m}^{-3}\text{)} \]
\[ T = \text{large-scale water flow (m}^3\text{ yr}^{-1}\text{)} \]
\[ f = \text{high latitude convection (m}^3\text{ yr}^{-1}\text{)} \]
\[ P = \text{particle flux (mol m}^{-2}\text{ yr}^{-1}\text{)} \]

\[ h = \text{high lat.} \]
\[ l = \text{low lat.} \]
\[ d = \text{deep} \]